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ON THE NUMBER OF CONNECTED COMPONENTS IN THE SPACE OF CLOSED NONDEGENERATE CURVES ON $\mathbf{S}^n$

B. Z. SHAPIRO AND M. Z. SHAPIRO

The main definition. A parameterized curve $\gamma\colon \mathbf{I} \to \mathbf{R}^n$ is called **nondegenerate** if for any $t \in \mathbf{I}$ the vectors $\gamma'(t), \dots, \gamma^{(n)}(t)$ are linearly independent. Analogously $\gamma\colon \mathbf{I} \to \mathbf{S}^n$ is called **nondegenerate** if for any $t \in I$ the covariant derivatives $\gamma'(t), \dots, \gamma^{(n)}(t)$ span the tangent hyperplane to $\mathbf{S}^n$ at the point $\gamma(t)$ (compare with the notion of n-freedom in [G]).

Fixing an orientation in $\mathbf{R}^n$ or $\mathbf{S}^n$ we call a nondegenerate curve γ right-oriented if the orientation on $\mathbf{S}^n$ induced by $\gamma', \dots, \gamma^{(n)}$ coincides with the given one and left-oriented otherwise.

Nondegenerate curves on $\mathbf{S}^n$ are closely related with linear ordinary differential equations of $(n+1)$ th order. Such an equation defines two nondegenerate curves on $\mathbf{S}^n \subset V^{(n+1)^*}$, where $V^{(n+1)^*}$ is the $(n+1)$-dimensional vector space dual to the space of solutions as follows. For each moment $t \in \mathbf{I}$ we choose the linear hyperplane in V^{n+1} of all solutions vanishing at t i.e. thus obtaining a unique curve in the projective space $\mathbf{P}^n$ as t varies. Raising it to $\mathbf{S}^n$ we obtain a pair of curves; both of them are right-oriented if n is odd and have opposite orientations if n is even (nondegeneracy follows from nonvanishing of its Wronskian).

A nondegenerate curve $\gamma\colon [0, 1] \to \mathbf{S}^n$ defines a monodromy operator $M \in \mathbf{GL}^+_{n+1}$ which maps $\gamma(0), \gamma'(0), \dots, \gamma^{(n)}(0)$ to $\gamma(1), \gamma'(1), \dots, \gamma^{(n)}(1)$.

The paper [K-O] contains a complete set of invariants for symplectic leaves of the second Gelfand-Dikki bracket; namely the leaves are enumerated by pairs consisting of a monodromy operator, and a connected component of the space of right-oriented curves in the sphere with the given monodromy operator.

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