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L FUNCTIONS FOR THE GROUP G_2

I. PIATETSKI-SHAPIRO, S. RALLIS, AND G. SCHIFFMANN

INTRODUCTION

The method of L functions is one of the major methods for analyzing automorphic forms. For example, the Hecke Converse Theorem gives an equivalence via the Mellin transform between holomorphic modular forms on the upper half plane and certain L functions associated to Dirichlet series, which have analytic continuation and functional equation. The classical theory of automorphic forms on the group GL_2 can be reinterpreted in terms of the spectral analysis of functions on the space $GL_2(k)\backslash GL_2(\mathbf{A})$ where $\mathbf{A} = \mathbf{A}_k$ is the adele ring of the number field k, ([W-1]). Using adelic language, Weil and Jacquet–Langlands have developed the Hecke Converse Theorem for GL_2 from a representation theoretic point of view ([W-2, J-L]). This leads to the problem of analyzing the class of automorphic representations of a reductive group, G. That is, we consider a $G(\mathbf{A})$ irreducible representation Π embedded in a suitable subspace of $G(k)\backslash G(\mathbf{A})$. By the general theory developed by Langlands ([L]) one can associate to Π a whole class of L functions parametrized by the finite-dimensional modules of an associated L group. The first two major questions that arise are whether these automorphic L functions have analytic continuation and functional equation. These questions are particularly relevant in that such automorphic L functions are closely tied to applications in number theory. For instance, it is expected from Langlands' philosophy that the L functions describing the arithmetic structure of certain algebraic varieties can be described in terms of automorphic L functions.

The problem of determining the analytic continuation and functional equation of general L functions associated to automorphic representations of reductive Lie groups and representations of the

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