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Stochastic processes with a multidimensional parameter, by M. Dozzi. Pitman Research Notes in Mathematics Series, Longman Scientific & Technical, Harlow, England, 1989, 195 pp., \$49.95. ISBN 0-582-03127-3

The general theory of stochastic processes with a multidimensional parameter began to emerge and develop in the 1970s, with the two pioneering works of R. Cairoli and J. B. Walsh [1], and of E. Wong and M. Zakai [4]. In order to understand this subject, let us recall that the general classical theory (see, for example, Dellacherie and Meyer's book [2]) deals with stochastic processes which are indexed by time, namely by a subset of the real line. Moreover, many of the basic tools of the theory depend on the total order structure of the parameter set. This is the case, for example, for the definition of a martingale, a Markov process, and the concept of stopping time. The fundamental difference between the classical theory and this one is the lack of a total order structure in the parameter set. In the multidimensional theory, the total order is replaced by the partial order induced by the Cartesian coordinates of the $\mathbf{R}^n$ space $(x_1, \dots, x_n) \le (y_1, \dots, y_n)$ when $x_i \le y_i$, for all $i = 1, \dots, n$.

The extensions of the different tools in this context yield very interesting results. For example, in the classical theory an $\mathscr{F}$-martingale $\{X_t\}$ can be defined equivalently by two points of view: either by writing $E[X_t/\mathscr{F}_s] = X_s$ for all $s \le t$, or by $E[X(s, t]/\mathscr{F}_s] = 0$, where $X(s, t] = X_t - X_s$ is the increment of the martingale on the interval $(s, t]$. These two equivalent formulas are generalized in the multidimensional case to two different concepts. The first one is called a martingale and the second one is called a weak-martingale. Moreover, following the kind of past you wish to consider, other types of martingales are obtained.

It turns out that all the kinds of martingales are necessary for the development of the theory: maximal inequalities, decompositions, optional sampling theorem, stochastic integrals, Ito's formula, stochastic differential equations, etc.

An interesting property of the filtrations is the following: Let $\{\mathscr{F}_t, t \in \mathbf{R}^n\}$ be an increasing family of σ-algebras of events and denote by $\mathscr{F}_t(C)$ the σ-algebra generated by the σ-algebras $\mathscr{F}_{t'}$,