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Theory of relations, by R. Fraïssé. Studies in Logic and the Foundations of Mathematics, vol. 118, North-Holland, Amsterdam, 1986, xii+397 pp., $55.25. ISBN 0-444-87865-3

What in the world is the theory of relations? I am not sure, but I know that I like it. What are the simplest relations on a set? Most mathematicians would include partial orders and linear orders. The theory of relations is concerned with these (as well as the more general n-ary relations) and basic notions such as embeddability and isomorphism. The subject originated with Hausdorff (*Grundzüze der Megenlehre*, 1914) and Sierpiński (*Leçons sur les nombres transfinis*, 1928).

One of my favorite topics is the theory of betterquasiorders. A quasiorder is a binary relation on a set which is reflexive and transitive, but not necessary antisymmetric. A quasiorder $\leq$ becomes a partial order if we mod out by the equivalence relation $\approx$ defined by $x \approx y$ iff $x \leq y$ and $y \leq x$. A good example of a quasiorder is to take the family of linear orders and put $L_1 \leq L_2$ iff there exists an embedding of L_1 into L_2, i.e., one-to-one mapping $f: L_1 \mapsto L_2$ such that for all x, $y \in L_1$ we have $x \leq_1 y$ iff $f(x) \leq_2 f(y)$.

One of the more interesting properties a quasiorder can have is that of being a wellquasiorder. A quasiorder $(X, \leq)$ is a wellquasiorder iff for any sequence $\langle x_n : n \in N \rangle$ of elements of X there exists some $n < m$ such that $x_n \leq x_m$. Clearly a wellquasiorder cannot have an infinite strictly descending sequence nor an infinite sequence of incomparable elements. Surpisingly these two properties characterize wellquasiorders. This is a nice exercise using Ramsey's theorem.

On of the first theorems about wellquasiorderings is due to Kruskal (1960). This theorem says that the finite trees are wellquasiordered under embeddability. A tree is partial order $(T, \leq)$ such that every initial segment, i.e., set of the form $\{y \in T : y \leq x\}$, is linearly ordered by $\leq$.

In 1948 Fräissé conjectured that the family of countable linear orders is wellquasiordered under the quasiorder of embeddability. Laver (1971) was able to prove this conjecture. One interesting