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FUNDAMENTAL SOLUTIONS AND HARMONIC ANALYSIS ON NILPOTENT GROUPS

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Our purpose here is to announce results in harmonic analysis related to a large class of hypoelliptic operators on arbitrary simply-connected nilpotent Lie groups. We find the asymptotic development of their fundamental solutions, both locally and at infinity, and study corresponding Riesz transforms and analogues of the Hardy–Littlewood-Sobolev theorem for fractional integration. Details will appear elsewhere (see [NRS]).

For linear partial differential operators $P = \sum_{|\alpha|\leq m} a_\alpha(x)D^\alpha$ on domains in $\mathbf{R}^n$, a classical approach to studying existence and regularity of solutions is the procedure of freezing coefficients: for each point x_0 one approximates the operator P by the constant coefficient operator $P_{x_0} = \sum_{|\alpha|=m} a_\alpha(x_0)D^\alpha$. The operator P_{x_0} is then a translation invariant operator on the group $\mathbf{R}^n$, and at least when P is elliptic, P_{x_0} can be inverted and its inverse studied by using the harmonic analysis of the Abelian Lie group $\mathbf{R}^n$. This is essentially the idea of the usual calculus of pseudo-differential operators. For nonelliptic operators, one cannot use this simple procedure, but starting with the work of [FoS] and [RoS] it is now known how to analyze certain hypoelliptic operators P by approximating them at each point by a left invariant differential operator on an appropriate nilpotent Lie group.

These operators all arise as polynomials in real vector fields, where the vector fields are assumed to satisfy the Hörmander [H] condition that the Lie algebra they generate spans the tangent space at each point. The method used is to model the algebra generated by these vector fields by the Lie algebras of certain nilpotent Lie groups. Further analysis allowed one to determine at least the approximate size of the fundamental solutions of these operators (see [NSW]). Thus the idea of approximating by nilpotent groups

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