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NOTES ON INVARIANT SUBSPACES

HARI BERCOVICI

ABSTRACT. The main purpose of this article is to give an approach to the recent invariant subspace theorem of Brown, Chevreau and Pearcy: Every contraction on a Hilbert space, whose spectrum contains the unit circle has nontrivial invariant subspaces. Our proof incorporates several of the recent ideas tying together function theory and operator theory.

1. INTRODUCTION

The Jordan structure theorem for finite matrices has been known now for over one hundred years, and its usefulness can hardly be overstated. It says that every square matrix A over the complex numbers $\mathbf{C}$ is similar to another matrix B (i.e., $B = XAX^{-1}$ for some invertible matrix X) which is a direct sum of Jordan cells. That is, B can be written in block form

$$B = \begin{bmatrix} B_1 & 0 & \cdots & 0 \\ 0 & B_2 & \cdots & 0 \\ \cdot & \cdot & \cdots & \cdot \\ 0 & 0 & \cdots & B_k \end{bmatrix}$$

and each B_i has the form

$$B_i = \begin{bmatrix} \lambda_i & 1 & 0 & \cdots & 0 \\ 0 & \lambda_i & 1 & \cdots & 0 \\ 0 & 0 & \lambda_i & \cdots & 0 \\ \cdot & \cdot & \cdot & \cdots & \cdot \\ 0 & 0 & 0 & \cdots & \lambda_i \end{bmatrix}$$

for some $\lambda_i \in \mathbf{C}$. The numbers $\{\lambda_1, \lambda_2, \dots, \lambda_k\}$ can be identified as the spectrum, or set of eigenvalues of A,

$$\begin{aligned}\{\lambda_1, \lambda_2, \dots, \lambda_k\} &= \{\lambda \in \mathbf{C}\colon \det(\lambda I - A) = 0\} \\ &= \{\lambda \in \mathbf{C}\colon \lambda I - A \text{ is not invertible}\}.\end{aligned}$$

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