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Introduction to superanalysis, by Felix Alexandrovich Berezin. Edited by A. A. Kirillov. Translated by J. Niederle and R. Kotecký. Mathematical Physics and Applied Mathematics, vol. 9, D. Reidel Publishing Company, Dordrecht, 1987, xii + 424 pp., \$109.00. ISBN 90-277-1668-4

Superalgebra, by F. A. Berezin, is a posthumous work, combining several of Berezin's preprints on Lie supergroups and superalgebras with fragments of a textbook on supermanifolds. Included also is an appendix by V. I. Ogievetsky on supersymmetry and supergravity.

At its foundational level, superanalysis is a simple generalization of commutative algebraic geometry. The basic notion is that of a $\mathbb{Z}_2$ -graded vector space, a vector space V with a decomposition into two subspaces: $V = V_0 \oplus V_1$. For $v \in V_i$, i is called the *parity* of v , and is denoted by $|v|$. If V and W are $\mathbb{Z}_2$ -graded vector spaces, then there is a supertwist map

$$\begin{aligned} V \otimes W &\rightarrow W \otimes V \\ v \otimes w &\rightarrow (-1)^{|w||v|} w \otimes v. \end{aligned}$$

A $\mathbb{Z}_2$ -graded algebra A is *supercommutative* if it satisfies the *usual* diagram for commutativity

$$\begin{array}{ccc} A \otimes A & \xrightarrow{T} & A \otimes A \\ \text{mult} & & \text{mult} \\ & A & \end{array}$$

where T is now the *supertwist*.

Thus supercommutative algebras, being in a sense commutative, inherit virtually unaltered the elementary theory of their commutative counterparts. The subject of superanalysis therefore organizes itself around the following themes:

1. Any superanalytic object will ultimately bear a strong resemblance to its classical analogue.
2. The understanding the first theme often requires more than a superficial understanding of the classical case, and this in itself makes the study of superanalysis worthwhile.

Berezin's book seems to resist these guiding principles. It conveys rather the impression that with the introduction of anticommuting coordinates, a new world of "supermathematics" appears.

The introductory chapter lays out the definitions of supermanifold, Lie superalgebra, integral and differential calculus on superspace. These definitions are repeated and expanded in later chapters.

Chapter 1 is entitled *Grassmann algebra*. By Grassmann algebra we mean an exterior algebra, $\Lambda_A(M)$, where A is a commutative algebra and M is a free A -module. If A is the algebra of smooth functions on a domain $U \subset \mathbb{R}^n$, and if a set of generators $\xi^1, \dots, \xi^m$ is given for M , then