BULLETIN (New Series) OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 19, Number 1, July 1988

IS OUR MATHEMATICS NATURAL? THE CASE OF EQUILIBRIUM STATISTICAL MECHANICS

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ABSTRACT. Is human 20th-century mathematics a natural or an arbitrary logical construct? Some insight into this ill-posed but fascinating question is gained from the study of mathematical physics. A number of ideas introduced by physical rather than mathematical necessity turn out later to be mathematically useful. (Equilibrium statistical mechanics has been a particularly rich source of such ideas.) In view of this the author argues that our mathematics may be much more arbitrary than we usually like to think.

1. Is our mathematics natural? The story goes that, when he reached heaven, Wolfgang Pauli requested to see his Creator, and asked Him to explain why the fine structure constant of electrodynamics has the value $\alpha \approx 1/137$. The Almighty went to the blackboard, and was busy writing formulae for a couple of hours. During that time Pauli sat, shaking his head and not saying a word. When finally the answer came: $\alpha^{-1} = 137.0359\ldots$, Pauli stood up, still shaking his head, took the chalk and pointed to an essential flaw in the calculation. I heard the story from Res Jost, and I wouldn't bet that it is completely authentic. Anyway, I think that many of us would like to ask some questions about physics and mathematics of Him who knows—when the opportunity arises. There are a number of obvious questions. For instance, about the consistency of mathematics: has He perhaps set up things, as Pierre Cartier suggests [**1**], so that the axioms of set theory are inconsistent, but a proof of contradiction would be so long that it could not be performed in our physical universe? Is this universe of ours the best of all possible worlds? Is it the only one of its kind, or could the fine structure constant be different from what it is? What kind of mathematics could be developed by intelligent beings living on a distant planet? Or in another universe with different physical laws?

Henri Poincaré once remarked that, for a question to make sense, one should be able to conceive of an answer which makes sense. This is not necessarily the case for the problems stated above. In fact, the problems which interest us most are often not easy to formulate satisfactorily. As a

Received by the editors January 13, 1988.

1980 *Mathematics Subject Classification* (1985 *Revision*). Primary 00A25; Secondary 82A05.

This paper is the 61st Josiah Willard Gibbs Lecture, presented by the author at the 94th annual meeting of the American Mathematical Society in Atlanta, Georgia on January 6, 1988. The manuscript was completed while the author was visiting the California Institute of Technology as a Fairchild Scholar. Permanent address: I.H.E.S., 91440 Bures-sur-Yvette, France.

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