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A NOTE ON CALDERÓN-ZYGMUND SINGULAR INTEGRAL CONVOLUTION OPERATORS

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The purpose of this note is to show that the notion of weak maximal function introduced in [**1**] (see also [**4**], where a similar notion is considered) can be used to obtain some new information on the Calderón-Zygmund singular integral convolution operator.

We will follow the notations of [**3**]. Let K be a kernel in $\mathbf{R}^n$ of class C^1 outside the origin satisfying

$$|K(x)| \leq C|x|^{-n}, \tag{1}$$

$$|\nabla K(x)| \leq C|x|^{-n-1}. \tag{2}$$

For $\varepsilon > 0$ and $f \in L^p(\mathbf{R}^n)$, $1 \leq p < \infty$, set

$$T_\varepsilon(f)(x) = \int_{|y|\geq\varepsilon} f(x-y)K(y)\,dy$$

and

$$T(f)(x) = \lim_{\varepsilon\to 0} T_\varepsilon(f)(x), \qquad T^*(f)(x) = \sup_{\varepsilon>0} |T_\varepsilon(f)(x)|.$$

We will assume that K satisfies the usual properties ensuring that the mapping $f \mapsto T^*(f)$ is of weak type $(1,1)$ and that $T(f)(x)$ makes sense for a.e. x.

The notation $L^{1,\infty}$ will stand for the space of weak L^1 functions, and if $\varphi \in L^{1,\infty}$ and B is a ball we write

$$||\varphi||^B_{1,\infty} = \sup_{\delta>0} \delta m(\{x \in B\colon\ |\varphi(x)| > \delta\})$$

for the weak L^1 "norm" of φ on B. If $B = \mathbf{R}^n$, we simply write $||\varphi||_{1,\infty}$.

The *weak maximal function* introduced in [1] is defined for $\varphi \in L^{1,\infty}$ by

$$M_w\varphi(x) = \sup_{x\in B} \frac{||\varphi||^B_{1,\infty}}{m(B)},$$

the supremum being taken over all balls centered at x. The notation $M_w^m\varphi$ stands for the function obtained by applying m times the operator M_w, whenever this makes sense. In [**1**] it was already pointed out that for any m there is a $\varphi \in L^{1,\infty}$ such that $M_w^j\varphi \in L^{1,\infty}$ for $j = 1, \dots, m$ but $M_w^{m+1}\varphi \notin L^{1,\infty}$. However, for $\varphi = Tf$, $f \in L^1$, the following holds:

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