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THE STEINBERG REPRESENTATION

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To Robert Steinberg on his 65th birthday

Introduction. Group representations occupy a sort of middle ground between abstract groups and transformation groups, i.e., groups acting in concrete ways as permutations of sets, homeomorphisms of topological spaces, diffeomorphisms of manifolds, etc. The requirement that the elements of a group act as linear operators on a vector space limits somewhat the complexity of the action without sacrificing the depth or applicability of the resulting theory. As in other areas of mathematics, study of linear phenomena may illuminate more general phenomena.

The widespread use of group representations in mathematics (as well as in physics, chemistry, ...) does not imply the existence of a single unified subject, however. Nor do practitioners always understand one another's language. Groups come in many flavors: finite, infinite-but-discrete, compact, locally compact, etc. Vector spaces may be finite or infinite dimensional; in the latter case there might be a Hilbert space structure and operators might be required to be unitary. The underlying scalar field may be complex, real, p-adic, finite, ... One can also make groups act on free modules over rings of arithmetic interest such as $\mathbf{Z}$.

Even the study of finite group representations, which probably came first historically, has become somewhat fragmented. Traditionally one considers representations of finite groups by $n \times n$ matrices with entries from $\mathbf{C}$. These are the "ordinary" representations. But in the late 1930s Richard Brauer began to show the usefulness of "modular" representations (with matrix entries lying in a field of prime characteristic) as a tool in the ordinary theory and in the structure theory of finite groups. There is now an active modular industry, with a life of its own, benefiting from recent innovations such as quivers and almost split sequences in the representation theory of finite dimensional algebras (which include group algebras). Study of "integral" representations is equally active, motivated by number-theoretic considerations or by questions raised by topologists about integral group rings of fundamental groups.

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