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Lectures on exponential decay of solutions of second-order elliptic equations: bounds on eigenfunctions of N-body Schrödinger operators, by Shmuel Agmon, Mathematical Notes, Vol. 29, Princeton University Press, Princeton, New Jersey, 1982, 118 pp., $10.50. ISBN 0-6910-8318-5

Square integrable eigenfunctions of the Schrödinger equation decay exponentially. More precisely, let

$$\tilde{H} = -\sum_{i=1}^{N} \frac{1}{2m_i}\Delta_i + \sum_{i<j} V_{ij}(x_i - x_j), \qquad x_i \in \mathbf{R}^\nu,$$

be the Schrödinger Hamiltonian for N particles interacting with real pairwise potentials $V_{ij}(x_i - x_j)$, where $V_{ij}(x_i - x_j) \to 0$ (in some sense) as $x_i - x_j \to \infty$ in $\mathbf{R}^\nu$. Separating out the center of mass ($\tilde{H}$ itself has only continuous spectrum) one obtains the operator

$$H = -\Delta + \sum_{i<j} V_{ij}(x_i - x_j),$$

where Δ denotes the Laplacian on $L^2(X)$, $X = \{x = (x_1, \ldots, x_N): \sum_{i=1}^N m_i x_i = 0\}$. If ϕ is an L^2 solution of $H\phi = E\phi$, and if E lies below the essential spectrum of H, then ϕ decays exponentially in the sense that there exist positive constants A and B for which $|\phi(x)| \leqslant Ae^{-B|x|}$. The phenomenon of exponential decay has long been recognized and was apparent already in Schrödinger's solution of the hydrogen atom, but is is only recently that a satisfactory mathematical theory for the problem has been developed.

There is a considerable chemical, physical, and mathematical literature on the subject, and we refer the reader to **[9, 7]**, and also the notes to Chapter XIII of **[14]**, for extensive historical and bibliographic information. Four general techniques have emerged.

(1) *Comparison methods* (see for example **[4, 5** and **3]**). These methods are based on the maximum principle for second order elliptic operators and are modelled, to a greater or lesser extent, on the standard proofs of such classical theorems of complex analysis as the Hadamard three-line theorem, the Phragmen–Lindelöf theorem, and so on.