BOOK REVIEWS

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Riemannian geometry, by Wilhelm Klingenberg, de Gruyter Studies in Mathematics, Vol. 1, Walter de Gruyter & Co., Berlin, 1982, x + 396 pp., $49.00. ISBN 3-1100-8373-5

The basic concepts of Riemannian geometry have become useful in a surprising variety of mathematical subjects. The terminology of manifolds, bundles, Riemannian metrics, and connections has become a *lingua franca* over much of partial differential equations, mathematical physics, and algebraic geometry, among other fields. This increasingly widespread use of the terminology and methods of geometry has tended to obscure the fact that Riemannian geometry as such is a subject with a quite precisely focused program, namely, to determine how the topology of a manifold is influenced by the local properties of its metric structure. These local properties are usually formulated in terms of curvature; this formulation is justified by the theorem of E. Cartan that the curvature tensor and its covariant derivatives of all orders at a point determine the formal Taylor expansion of the metric at that point. The program of obtaining global topological information from local metric information actually applies only to complete Riemannian manifolds, i.e., those that are complete as metric spaces when the distance between points is defined to be the infimum of the Riemannian-metric arc length of curves joining the two points. This restriction to complete manifolds, long a standard one in the subject, has been given explicit justification by a result of M. Gromov that a noncompact manifold admits noncomplete Riemannian metrics with essentially arbitrary curvature behavior.

A complete Riemannian manifold has the property that the geodesics (curves that locally minimize distance) emanating from a fixed but arbitrary point cover, taken together, the whole manifold. It follows that understanding the behavior of these geodesics completely would yield total information about the global structure of the manifold itself. This rather vague description can be given real mathematical substance; and from this general idea, a body of results has been obtained that is the central part of Riemannian geometry. These results are the subject of this book.

To describe the contents of the book in more detail, it is necessary to recall the concept of Riemannian sectional curvature. This concept is a natural