

THE DYNAMICAL SYSTEMS APPROACH TO DIFFERENTIAL EQUATIONS

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This harmony that human intelligence believes it discovers in nature —does it exist apart from that intelligence? No, without doubt, a reality completely independent of the spirit which conceives it, sees it or feels it, is an impossibility. A world so exterior as that, even if it existed, would be forever inaccessible to us. But what we call objective reality is, in the last analysis, that which is common to several thinking beings, and could be common to all; this common part, we will see, can be nothing but the harmony expressed by mathematical laws.

H. Poincaré, *La valeur de la science*, p. 9

...ignorance of the roots of the subject has its price—no one denies that modern formulations are clear, elegant and precise; it's just that it's impossible to comprehend how any one ever thought of them.

M. Spivak, *A comprehensive introduction to differential geometry*

INTRODUCTION

The mathematical subject we call dynamical systems was fathered by Poincaré, developed sturdily under Birkhoff, and has enjoyed a vigorous new growth for the last twenty years. As I try to show in Chapter I, it is interesting to look at this mathematical development as the natural outcome of a much broader theme which is as old as science itself: the universe is a system which changes in time.

Every mathematician has a particular way of thinking about mathematics, but rarely makes it explicit. Yet such perspectives, in any particular field, can be of great value to nonexperts who must apply the results of the field, or who want to learn them. In some fields the points of view of its practitioners cluster around several comparatively clearly defined concepts and insights. Dynamical systems theory is one of them; the basic concepts go back to Poincaré. The main thrust of Chapters I and II is to explain, rather discursively, some of

Received by the editors December 14, 1983.

1980 *Mathematics Subject Classification*. Primary 00A25, 00A99, 01A45, 01A55, 01A60, 34A40, 34C35, 34D10, 34D30, 35B05, 35B35, 35B40, 35B50, 35B30, 35K55, 54H20, 58F25, 58F40, 58F12, 58F10, 58D25, 58D07, 46A40, 46E10, 47H07, 47H20, 92A15, 92A17; Secondary 06F30, 35B65, 58F13, 35J60, 46E05, 92-03, 90-03, 90A16.

¹ Supported in part by National Science Foundation and Institute for Advanced Study.