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SUBDIFFERENTIABILITY SPACES AND NONSMOOTH ANALYSIS

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We use the words "subdifferentiability space" as a collective name for classes of Banach spaces characterized by subdifferentiability properties of l.s.c. nonconvex functions defined on them. The classes were introduced by analogy with Asplund (differentiability) spaces [**1**], but the initial impulse came from nonsmooth analysis where one specific class had appeared quite naturally [**2**]. The purpose of the note is to introduce four classes of subdifferentiability spaces, describe their properties and the role of one of the classes in nonsmooth analysis.

1. All spaces are assumed Banach and the functions extended real-valued. For a function f on X we set

$$\operatorname{dom} f = \{x \mid |f(x)| < \infty\},$$
$$U(f,z,\delta) = \{x \in \operatorname{dom} f \mid \|x - z\| < \delta,\ |f(x) - f(z)| < \delta\},$$
$$d^- f(z;h) = \liminf_{\substack{u \to h \\ t \searrow 0}} t^{-1}(f(z+tu) - f(z))$$

($z \in \operatorname{dom} f$); B, B^* denote unit balls in X, X^*.

DEFINITION 1. Let f be a function on X, $\epsilon > 0$, and $z \in \operatorname{dom} f$. We denote by $\varphi_\epsilon^-(z)$ the set of all $x^* \in X^*$ such that

$$\liminf_{\|h\| \to 0} \|h\|^{-1}(f(z+h) - f(z) - \langle x^*, h\rangle) \geq -\epsilon,$$

and by $\partial_\epsilon^- f(z)$ the set of all $x^* \in X^*$ such that

$$\langle x^*, h\rangle \leq d^- f(z;h) + \epsilon\|h\|;$$

$\varphi_\epsilon^- f(z)$ and $\partial_\epsilon^- f(z)$ will be respectively called the *Fréchet* and the *Dini ϵ-subdifferential* of f at z.

If $z \notin \operatorname{dom} f$, we set $\varphi_\epsilon^- f(z) = \partial_\epsilon^- f(z) = \varnothing$.

DEFINITION 2. X is a *subdifferentiability* (*weak subdifferentiability*) space or S-space (WS-space) if for any $\epsilon > 0$ every l.s.c. function f on X is Fréchet (Dini) ϵ-subdifferentiable on a dense subset of $\operatorname{dom} f$.

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