## THE INVARIANT THEORY OF BINARY FORMS

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Dedicated to Mark Kac on his seventieth birthday

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1. Introduction. Like the Arabian phoenix rising out of its ashes, the theory of invariants, pronounced dead at the turn of the century, is once again at the forefront of mathematics. During its long eclipse, the language of modern algebra was developed, a sharp tool now at long last being applied to the very

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