FUCHS AND THE THEORY OF DIFFERENTIAL EQUATIONS

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Introduction. Lazarus Immanuel Fuchs (1833–1902) was the leading theorist of differential equations in the 1860s and 1870s, and, with Frobenius and Schwarz, a principal member of the 'second generation' of Berlin mathematicians. He obtained his *Habilitation* from Berlin University in 1865 and was eventually called back to succeed Kummer in 1884. His work can profitably be seen as an attempt to impose upon the inchoate world of differential equations the conceptual order of the emerging theory of complex functions. As well as being the architect of the rigorous modern theory of linear equations, he raised many questions which were taken up by his contemporaries and provided an interesting battleground for the schools of invariant theory and transformation group theory. Most famously his work inspired Poincaré to create the theory of Fuchsian functions and Fuchsian groups. Through his work and his career we can gain an insight into the state of mathematics in the second half of the nineteenth century, as the original work of Weierstrass was built up into an imposing intellectual edifice.

Differential equation theory before 1866 and Fuchs' first contribution. It took Fuchs some time to find his true area of interest as a mathematician. His doctoral thesis, written under Kummer's supervision, was on lines of curvature on a surface. Then he wrote two papers on number theory concerning the periods of the nth roots of unity when n is not prime. But Kummer's influence was shortly to be outweighed by that of another. Weierstrass had been in Berlin as a professor at the Industrial Institute (the Gewerbeschule) since 1856, and in July 1864 he became professor at Berlin University. He had by then been lecturing at the University for some time. In the summer of 1863 he had lectured on Abelian functions and developed his theory of linear differential equations. This caught Fuchs' interest, and he then began to study the subject that occupied him for the rest of his life. His relationship with Weierstrass seems to have been close, and by 1870 he was writing to Casorati that he considered himself a pupil of Weierstrass (quoted in Neuenschwander [**1978b**, p. 46]).

Differential equations had been studied from various points of view by the 1860s, and we may distinguish two strands. One deals with particular equations, such as Legendre's equation (defined below) and its generalizations to