

THE INFLUENCE OF ELASTICITY ON ANALYSIS: THE CLASSIC HERITAGE

BY C. TRUESDELL

Dedicated to J. L. Ericksen on his 60th birthday

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Most mathematicians have an idea of the influence of hydrodynamics and electromagnetism on the theory of complex functions and harmonic potentials. The influence of elasticity is less well known. Elasticity led to a vast range of mathematical problems involving linear algebra, differential geometry, ordinary and partial differential equations (mostly nonlinear), elliptic functions, and the calculus of variations.

1. The catenary: twin solutions, and the calculus of variations. The contest of 1690 to find the catenary curve is described in all histories of mathematics. The body treated is a chain or rope without stiffness. The mathematical model for this body is a plane curve. This curve satisfies differential equations expressing the requirement that the resultant force and torque on each part of the body be zero. As early as 1675 Hooke had stated in an anagram, "as hangs the flexible line, so but inverted will stand the rigid arch", but Hooke was not a mathematician and could not prove anything. His statement applies nevertheless to Leibniz's differential equation for the catenary: If that equation has a solution bellied downward for the points considered, it also has one bellied upward

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