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The origins of Cauchy's rigorous calculus, by Judith V. Grabiner, MIT Press, Cambridge, 1981, viii + 252 pp., $30.00.

Équations différentielles ordinaires: *Ordinary differential equations*, by Augustin Louis Cauchy, Études Vivantes, Paris, and Johnson Reprint Corporation, New York, 1981, lviii + 146 pp., $24.50.

Calculus in 1800 was in a curious state. There was no doubt that it was correct. Mathematicians of sufficient skill and insight had been successful with it for a century. Yet no one could explain clearly why it worked. To be sure, experts would probably have agreed that some notion of "limit" lay behind derivatives, and of course integrals were defined as antiderivatives and thus raised no separate questions. But the discussions of the foundational issues had been desultory and inconclusive. Students by and large were not instructed in calculus, they were initiated into it. If they were gifted with the right insight, practice would then give them an intuitive feeling for the right results. The motto of the period, attributed (perhaps wrongly) to D'Alembert, was *Allez en avant, et la foi vous viendra*: Go forward, and faith will come to you.

Still, there was a nagging feeling that something should be done. There were occasional disagreements, like that over the vibrating string, that were hard to bring to a clear resolution. Besides, mathematicians still remembered the tradition of proof that was their proud inheritance from the Greeks. To establish something "in the style of geometry" was a byword for establishing it beyond doubt. Particularly galling was the fact that Archimedes had established some "calculus" results in exactly that style. It seemed in fact that every single area value or tangent slope computed by calculus could be similarly justified. But no one wanted to do such justifications, because they were long, tedious, and (worst of all) apparently unrelated to the intuition behind the calculus. Lagrange had been concerned with justifying calculus for over twenty years, but his major efforts had rested on the formal use of power series and were not satisfactory.

Then came Cauchy. It is hardly enough to say merely that he *solved* the problems; he showed that there *weren't* any problems. Seldom has there been such good reason to say, with Boileau,

> Ce que l'on conçoit bien s'énonce clairement,
> Et les mots pour le dire arrivent aisément.

For Cauchy was not at all the type of scholar who ponders and polishes his work for years. Throughout his career he wrote almost two papers a month. His last submission to the Académie des Sciences, less than three weeks before he died, ends with the words, "I shall explain this at greater length in a memoir to follow." Called upon to lecture on calculus, he merely presented the prescribed topics as best he could. But his best was so illuminating that the