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The Hardy-Littlewood method, by R. C. Vaughan, Cambridge Tracts in Mathematics, vol. 80, Cambridge University Press, Cambridge, 1981, xii + 172 pp., $34.50.

A few years ago I heard a prominent algebraic number theorist exclaim: "What, the Hardy-Littlewood method is still alive? I thought it had been dead long ago". The book under review shows that the method is alive and well!

Let $\mathfrak{F}\colon \mathfrak{D} \to \mathbf{Z}$ be a map into the integers assuming each value at most finitely often. The number $N = N(\mathfrak{F}, \mathfrak{D})$ of zeros of $\mathfrak{F}$ is the constant term of the formal series

$$F(z) = \sum_{\mathbf{x} \in \mathfrak{D}} z^{\mathfrak{F}(\mathbf{x})}.$$

Assuming that F is analytic in the disk $|z| < 1$ with the possible exception of $z = 0$, we may invoke Cauchy's integral formula to obtain

$$N = \frac{1}{2\pi i} \int_C z^{-1} F(z)\, dz, \tag{1}$$

where C is a circle centered at 0 with radius $\rho < 1$. What is surprising is not this formula, but the way in which the integral on the right may often be evaluated or approximated so as to give information about diophantine problems.

Hardy and Ramanujan [**1918**] used this integral formula to obtain an asymptotic relation for the partition function, and to deal with the number of representations of integers by sums of squares. More generally, in a series of papers beginning in 1920, Hardy and Littlewood applied the formula to Waring's problem, i.e. the representation of integers n by sums of nonnegative kth powers:

$$n = x_1^k + \cdots + x_s^k. \tag{2}$$

Here $\mathfrak{D} = \mathbf{Z}^+ \times \cdots \times \mathbf{Z}^+$ where $\mathbf{Z}^+$ are the nonnegative integers, $\mathfrak{F}(\mathbf{x}) = \mathfrak{F}(x_1, \ldots, x_s) = x_1^k + \cdots + x_s^k - n$, and $N = N(k, s, n)$. Hardy and Ramanujan noted that in the case $k = 2$, i.e. in the case of squares, the integrand in (1)