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SINGULAR CONVOLUTION OPERATORS ON THE HEISENBERG GROUP[1]

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1. Statement of results and outline of method. The purpose of this note is to announce results dealing with convolution operators on the Heisenberg group. As opposed to the well-known situation where the kernels are homogeneous and C^∞ away from the origin, the kernels we study are homogeneous but have singularities on a hyperplane. Convolution operators with such kernels arise in the study of the $\bar{\partial}$-Neumann problem, as we indicate below. The main feature of our study will be an analysis which has no direct analogue in the case of the usual (abelian) convolution operators, but is instead based on the noncommutative Fourier analysis of the Heisenberg group.

Let $\mathbf{H}^n$ denote the Heisenberg group, the Lie group with underlying manifold $\mathbf{C}^n \times \mathbf{R}$ and multiplication $(z, t) \cdot (z', t') = (z + z', t + t' + 2\operatorname{Im} z \cdot \bar{z}')$, where $z \cdot \bar{z}' = \Sigma z_j \bar{z}_j'$. $\mathbf{H}^n$ possesses dilations: let $D_r(z, t) = (r^2 z, rt)$. A function f is homogeneous of degree k if $f \circ D_r = r^k f$, and there is a dual notion for distributions. Suppose K' is a homogeneous distribution of degree $-2n - 2$ which agrees with a function away from the origin. Assume that this function is smooth on $\mathbf{H}^n - \{0\}$, or more generally that it satisfies an L^1-Dini condition there. Then it is known [**3**, Theorem 2.1] that the convolution operator A': $C_0^\infty(\mathbf{H}^n) \dashrightarrow C^\infty(\mathbf{H}^n)$ given by $A'f = f * K'$ extends to a bounded operator from L^p to L^p for $1 < p < \infty$ if it is bounded on L^2. Here $*$ denotes group convolution.

It is our intention to study more singular convolution operators. Thus, let $J(z)$ be a homogeneous distribution of degree $-2n$ on $\mathbf{C}^n$, which agrees with a smooth function away from the origin, and define the distribution K on $\mathbf{H}^n$ by $K(z, t) = J(z)\delta(t)$. Here $\delta(t)$ is the Dirac delta function in the t variable. Then K is homogeneous of degree $-2n - 2$ and we assert

THEOREM. *The operator A: $C_0^\infty(\mathbf{H}^n) \longrightarrow C^\infty(\mathbf{H}^n)$ given by $Af = f * K$ extends to a bounded operator from L^p to L^p for $1 < p < \infty$.*

The analogue of the theorem for Euclidean convolution is immediate, since one can convolve on each hyperplane t = constant separately. No such argument is available for $\mathbf{H}^n$. Our strategy in proving the theorem is to use

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