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Function theory in the unit ball of $\mathbf{C}^n$, by Walter Rudin, Grundlehren der mathematischen Wissenschaften, Band 241, Springer-Verlag, Berlin and New York, 1980, xiii + 436 pp., $39.00.

0. Introduction. It has often been remarked that the subject of real analysis had a fallow period during the 1930's and 1940's. Some have laid the blame, perhaps unjustly, at the feet of Hardy and Littlewood. It is said that they could see little interest in doing the theory of one variable with the additional clap-trap of multi-indices. So a lot of time and energy was instead expended in constructing exotic counterexamples and exploring remote corners of the theory of one variable. Immediate contradictions to what I have just said spring to mind: Zygmund, Marcinkiewicz, Saks, Wiener, Bochner, and many others did powerful and significant work during this period. But it is safe to say that while algebra, algebraic topology, and algebraic geometry were developing very rapidly from 1930–1950, real analysis was not making the (what by now seems) obvious move into the theory of several variables.

In retrospect, it is easy to understand how this recession in real analysis came about. The differences between real analysis of one and several variables–subellipticity, propagation of singularities, the existence of singular integrals, the failure of the multiplier problem for the ball, the connections between covering theorems and the boundedness of integral operators, restriction theorems for the Fourier transform, the theory of currents and the solution of the general Plateau problem, etc.–all lie very deep. It is amazing that *anyone* discovered these phenomena, much less mathematicians who believed that there were no phenomena to discover. Hardy and Littlewood could not have conceived what is now painfully clear: that $\mathbf{R}^1$ is the exceptional dimension, $\mathbf{R}^3$ is typical, and $\mathbf{R}^2$ is some intermediate bastardization.

Complex analysis has enjoyed rather a different history. Hartogs discovered quite early (1906) the phenomenon of domains of holomorphy. Recall that a holomorphic function of several complex variables is one which is holomorphic (in the one-variable sense) in each variable separately. Then Hartogs's result is

THEOREM 1 (HARTOGS). *Let*

$$\mathbf{C}^2 \supseteq \Omega \equiv \{|z_1| < 1, \quad |z_2| < 1\} \setminus \{|z_1| \leqslant 1/2, \quad |z_2| \leqslant 1/2\}.$$

Let $f\colon \Omega \to \mathbf{C}$ *be holomorphic. Then there is a holomorphic*

$$\hat{f}\colon \{|z_1| < 1, \quad |z_2| < 1\} \to \mathbf{C}$$

such that $\hat{f}|_\Omega = f$.

PROOF. For each fixed z_1, $|z_1| < 1$, write f as a Laurent series in z_2,

$$f(z_1, z_2) = \sum_{n=-\infty}^{\infty} a_n(z_1) z_2^n,$$