

of current research. The reviewer was particularly happy to see the inclusion of the next topic: plane cubic curves. This is dealt with in a long chapter, culminating in a classification of the singular and nonsingular cubics in  $PG(2, q)$ . There follow interesting chapters on plane  $(k: n)$ -arcs and blocking sets, with a final chapter of detailed analysis of  $PG(2, q)$  for small values of  $q$ .

One very helpful feature of the format of the text is a concluding section of each chapter, entitled "notes and references". Here the credits for the work of each preceding section are given, thus freeing the main text of a plethora of individual references, which would hamper the smooth flow of the development. The book is essentially self-contained and serves well the dual purpose of a good reference source for the expert and a vehicle of self-study for the newcomer to a fascinating but relatively unknown subject.

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*Translation planes*, by Heinz Lüneburg, Springer-Verlag, Berlin and New York, 1980, ix + 278 pp., \$29.80.

As the author says in his foreword, infinite translation planes are not completely disregarded but finite ones form the main theme of the book. Finite projective and affine planes have interest for mathematicians in many other areas. First, they are important for combinatorialists, including people in design of experiments and in coding theory. Many combinatorial structures have natural representations associated with finite projective planes. Projective planes (finite or infinite) have obvious significance for the foundations of geometry. If one were to look for the field of interest whose adherents were most likely to have a strong side interest in projective planes a strong candidate would be the field of finite groups. Many of the early developments in projective planes came from people working on nonassociative division rings (semifields).

The known finite projective (or affine) planes are closely related to translation planes.

One way of looking at the Euclidean plane is to identify the points with elements  $(x, y)$  of a two-dimensional vector space over the reals. The lines through the origin are one-dimensional vector spaces; the other lines are translates of the lines through the origin. This book develops the following analogous way of representing a translation plane: Let  $V$  be a vector space which is a direct sum of two copies of a subspace  $V_1$  so that  $V = V_1 \oplus V_1$ . Each element of  $V$  can then be represented by an ordered pair  $(X, Y)$ ,  $X, Y \in V_1$ . If  $V_1$  has dimension  $r$  then  $V$  has dimension  $2r$  and the sets of vectors for which  $X = 0$  or for which  $Y = 0$  are  $r$ -dimensional subspaces isomorphic to  $V_1$ . A *spread* defined on  $V$  is a class of  $r$ -dimensional subspaces (the *components* of the spread) such that each nonzero element belongs to exactly one component and  $V$  is the direct sum of each pair of distinct