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A CHARACTERIZATION OF HEREDITARY RINGS OF FINITE REPRESENTATION TYPE

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In this note we announce a characterization of hereditary rings R of finite representation type in terms of a sequence of partial Coxeter functors associated to R.

The methods we use are theory of almost split sequences [1], partial Coxeter functors [2], [3], [5], representations of species [7], and some ideas of Ringel [8, §6].

We recall from [7] that a *species* $\mathcal{M} = (F_i, {}_iM_j)_{i,j\in I}$ is a finite set of division rings F_i and $F_i - F_j$ bimodules ${}_iM_j$. Throughout we will suppose $\mathcal{M}$ is a species such that ${}_iM_j \neq 0$ implies ${}_jM_i = 0$ for $i \neq j$. $\mathcal{M}$ is called *finite dimensional* if the dimensions

$$d_{ij} = \dim({}_iM_j)_{F_j}, \qquad d_{ji} = \dim_{F_i}({}_iM_j)$$

are finite for $i \neq j$. We denote by $\mathfrak{R}(\mathcal{M})$ the category of all finite dimensional right representations of $\mathcal{M}$ (see [6], [8]). From $\mathcal{M}$ we derive an *oriented valued graph* $(\Gamma, \mathbf{d}, \Omega)$ (not necessarily symmetrizable) with valued edges

$$\underset{i}{\bullet} \xrightarrow{(d_{ij},\, d_{ji})} \underset{j}{\bullet}$$

precisely when ${}_iM_j \neq 0$. Given source (resp. sink) k in $(\Gamma, \mathbf{d}, \Omega)$ we define a species $\mathcal{M}^k = (F_i, {}_iN_j)_{i,j\in I}$ by taking

$${}_iN_j = \begin{cases} {}_kM_i^{i} = \operatorname{Hom}_{F_i}({}_kM_i, F_i) & \text{for } j = k \quad (\text{resp.} = {}_jM_k^{j} \text{ for } i = k) \\ {}_iM_j \quad \text{for} \quad j \neq k & (\text{resp. for } i \neq k). \end{cases}$$

If $\mathcal{M}$ is a finite dimensional species and k is a sink in $(\Gamma, \mathbf{d}, \Omega)$ we can define a pair of *Coxeter functors*

$$\mathfrak{R}(\mathcal{M}) \underset{S_k^-}{\overset{S_k^+}{\rightleftarrows}} \mathfrak{R}(\mathcal{M}^k)$$

having following properties:

(c_1^+) S_k^- is left adjoint to S_k^+.

(c_2^+) Suppose $\mathbf{X} = (X_i, {}_j\varphi_i)$ is indecomposable in $\mathfrak{R}(\mathcal{M})$. Then

(i) $S_k^+\mathbf{X} = 0$ if and only if $\mathbf{X} \cong \mathbf{F}_k$, where $\mathbf{F}_k$ has F_k on kth coordinate and zeros otherwise,

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