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Recursion-theoretic hierarchies, by Peter G. Hinman, Perspectives in Mathematical Logic, Springer-Verlag, Berlin, Heidelberg, New York, 1978, xii + 480 pp.

The turn of the century saw (amongst other things) the beginnings of Descriptive Set Theory. Faced with the increasing use of the all-powerful methods of Cantorian set theory and, in particular, the Axiom of Choice, mathematicians such as Baire, Borel, and Lebesgue began to develop more constructive approaches to analysis. Out of such investigations came the definition of the Borel sets, Souslin's theorem that a set of real numbers is Borel iff both it and its complement are analytic, the result that analytic sets are measurable and have the Baire property, and so on. Inherent in the notions of Borel set and analytic set (etc.) is the notion of hierarchy. Roughly speaking, a hierarchy is a classification of certain mathematical objects into levels, indexed by natural numbers, or possibly transfinite ordinal numbers as well. Objects appearing in levels low in the hierarchy are somehow more simple than those in higher levels: passage up through the hierarchy represents a gradually increasing complexity of the objects covered. For example, consider the Borel sets of real numbers. The standard definition of this class is that it is the smallest class of sets which contains all intervals and is closed under the formation of complements and countable unions. We can impose a hierarchy on this class by putting into the α-th level all sets which require a sequence of α applications of complementation and countable union to families of intervals for their construction. This hierarchy has precisely ω_1 levels, the level of any particular Borel set providing a measure of its complexity as a Borel set. Using this hierarchy we can, for instance, prove results about Borel sets by induction on the levels of the hierarchy.

Such hierarchies form a large part of the subject matter of this book. That this is so in a book published in the series "Perspectives in Mathematical Logic" (i.e. that the study of such hierarchies has fallen into the domain of the mathematical logician) arises from the marriage between classical Descriptive Set Theory and the developments in Logic which took place in the 1930s (and onwards). Logicians such as Church, Kleene, Turing, and Mostowski were looking at the notions of algorithmically calculable function, one function being recursive in another, the arithmetical and analytic hierarchies, and various sorts of definability in formal languages. By the mid 1950s, it became clear that classical Descriptive Set Theory and the above mentioned parts of Recursion Theory are really special cases of a single general theory of definability, this realisation being formally acknowledged by Addison's announcements in BAMS **61** (1955), 75; 171–172 (*Analogies in the Borel, Lusin, and Kleene hierarchies*. I, II). This general definability theory forms the starting point of the book.

The author is clearly addressing himself to more advanced students, a reasonable knowledge of analysis, topology, measure theory, set theory and