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THE DUALITY OPERATION IN THE CHARACTER RING OF A FINITE CHEVALLEY GROUP

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It is possible (as in [4]) to define a duality operation $\zeta \longrightarrow \zeta^*$ in the ring of virtual characters of an arbitrary finite group with a split (B, N)-pair of characteristic p. Such a group arises as the fixed points under a Frobenius map of a connected reductive algebraic group, defined over a finite field [1]. This paper contains statements of several general properties of the duality map $\zeta \longrightarrow \zeta^*$ and two related operations (see §§2 and 4). The duality map $\zeta \longrightarrow \zeta^*$ generalizes the construction in [2] of the Steinberg character, and interacts well with the organization of the characters from the point of view of cuspidal characters (§6). It is hoped that there is also a useful interaction with the Deligne-Lusztig virtual characters $R_{\mathrm{T}}^{\mathrm{G}}\theta$. Partial results have been obtained in this direction (§5). Detailed proofs will appear elsewhere.

1. Let G be a finite group with split (B, N)-pair of characteristic p. Let (W, R) be the Coxeter system, and let $P_J = L_J V_J$ be the standard parabolic subgroup corresponding to $J \subseteq R$, with $V_J = O_p(P_J)$ (see [3] for definitions and notations). Let char(G) denote the ring of virtual characters of G, and Irr(G) the set of irreducible characters of G, all taken in the complex field. For $J \subseteq R$ and $\zeta \in \text{char}(G)$ define

$$\zeta_{(P_J/V_J)} = \Sigma(\zeta, \tilde{\lambda}^G)_G \lambda \tag{1.1}$$

where $\sim$ denotes extension to P_J via the projection $P_J \longrightarrow L_J \cong P_J/V_J$, and the sum is over all $\lambda \in \text{Irr}(L_J)$. Let $\zeta_{(P_J)} = \zeta_{(P_J/V_J)}{}^{\sim}$. The duality map is then defined by:

1.2 DEFINITION. $\zeta^* = \Sigma_{J \subseteq R}(-1)^{|J|}\zeta_{(P_J)}{}^G$, for all $\zeta \in \text{char}(G)$.

2. The truncation map $\zeta \longrightarrow \zeta_{(P_J/V_J)}$ and the map $\lambda \longrightarrow \tilde{\lambda}^G$ behave in much the same way as ordinary restriction and induction. The following basic properties follow directly from the structure theorems [3].

2.1 FROBENIUS RECIPROCITY. *Let* $\zeta \in \text{char}(G)$ *and* $\lambda \in \text{char}(L_J)$. *Then*

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