## RATIO IN EARLY GREEK MATHEMATICS

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# **Notes**

Referencing. The following abbreviated titles are used for books to which frequent page-references are made:

Burkert, LS = Lore and science in ancient Phythagoreanism.

Euclid, EE = Elements, ed. and tr. T. L. Heath, 3 vols.

Heath, HGM = A history of Greek mathematics, 2 vols.

Knorr, EEE = The evolution of the Euclidean elements.

Thomas, SGM = Selections illustrating the history of Greek mathematics, 2 vols.

van der Waerden, SA = Science awakening.

Other references are by author and date. Full bibliographical details are listed at the end of the paper.

### 1. Introduction.

SOCRATES: Are there not two kinds of arithmetic, that of the people and that of philosophers?... And how about the arts of reckoning and measuring as they are used in building and in trade when compared with philosophical geometry and elaborate computations—shall we speak of each of these as one or two?

PROTARCHUS: I should say that each of them was two.

Plato, Philebus 56D-57E.

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