

RECURSIVELY ENUMERABLE SETS AND DEGREES

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TABLE OF CONTENTS

Introduction

Chapter I. The relation of the structure of an r.e. set to its degree.

1. Post's program and simple sets.
2. Dominating functions and quotient lattices.
3. Maximal sets and high degrees.
4. Low degrees, atomless sets, and invariant degree classes.
5. Incompleteness and completeness for noninvariant properties.

Chapter II. The structure, automorphisms, and elementary theory of the r.e. sets.

6. Basic facts and splitting theorems.
7. Hh -simple sets.
8. Major subsets and r -maximal sets.
9. Automorphisms of \mathcal{E} .
10. The elementary theory of \mathcal{E} .

Chapter III. The structure of the r.e. degrees.

11. Basic facts.
12. The finite injury priority method.
13. The infinite injury priority method.
14. The minimal pair method and lattice embeddings in \mathbf{R} .
15. Cupping and splitting r.e. degrees.
16. Automorphisms and decidability of \mathbf{R} .

Introduction. G. E. Sacks has remarked that recursion theory is the heart of logic, and recursively enumerable sets form the soul of recursion theory. Although some might challenge these claims, it is clear that recursively enumerable sets have played an important role in logic beginning with the first undecidability results of Gödel [Gö1], Church [Ch] and Rosser [Rs]. Furthermore, the notion of a recursively enumerable set rather than that of a recursive (i.e., computable) function has proved to be the fundamental concept in attempts to generalize classical recursion theory to more general settings, such as admissible ordinals [Sh4], [Le6], or higher types [Sa9].

A subset A of ω (the set of nonnegative integers) is *recursive* (also called *decidable* or *computable*) if there is an algorithm for determining whether a

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