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BULLETIN OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 84, Number 3, May 1978
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Elliptic functions and transcendence, by David Masser, Lecture Notes in Math., vol. 437, Springer-Verlag, Berlin, Heidelberg, New York, xii + 143 pp., $7.80.

I. A history. The original proof of the transcendental nature of the number e by Hermite in 1873 was based on a delicate scheme of rational approximations which seemed to be applicable only to the exponential function. In this light one may view with a sympathetic eye Hermite's pessimism toward the problem of the transcendental nature of π, as he openly states in a letter to Borchardt (Crelle, vol. 76, p. 342): "Que d'autres tentent l'entreprise, nul ne sera plus heureux que moi de leur succès, mais croyez-m'en, mon cher ami, il ne laissera pas que de leur en coûter quelques efforts." A few years later in 1882 Hermite would be amazed by the remarkable simplicity of Lindemann's proof of the transcendentality of π based on Euler's identity $e^{\pi i} = -1$ and on Hermite's earlier ideas. This episode marks the exalting birth of the theory of transcendental numbers and was to represent the only significant contribution for some time. What followed in the next quarter of a century was no more than a generalization of ideas and a simplification of methods, first in the hand of Weierstrass who saw that the method of Hermite and Lindemann could be made to yield a proof of the algebraic independence of the values of the exponential function at distinct algebraic points; this was followed by technical simplifications by Gordan, Hilbert and Hurwitz.

By the end of the nineteenth century it was generally believed that the main arithmetical properties of the exponential function were well understood; there were good reasons for this. For one, the work of Kummer on cyclotomic extensions had been around for more than half a century, even though his methods were beginning to be forgotten; the work of Kronnecker on complex multiplication was being brought to completion. One knew well that the values taken by the exponential function $e^{2\pi iz}$ at the rational points on the projective line $\mathbf{P}^1(Q)$ were values at *special points*, i.e. they generate abelian extensions of the rationals and all such extensions arose in this way. One may surmise that in 1900 Hilbert, being thoroughly familiar with these properties of the exponential function after the manner of his Bericht, would have present in the back of his mind these results when formulating his Seventh Problem on the arithmetical nature of numbers of the form α^{β} and in particular of $2^{\sqrt{2}}$, and in his Twelfth Problem concerning the search for automorphic forms whose values at *special points* of certain moduli varieties would generate algebraic extensions of number fields with special Galois properties.