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1. Introduction. The early work of Klein [**28**] and Poincaré [**53**] on the uniformization of algebraic curves (compact Riemann surfaces) was based on the so-called *continuity method*. After a few years, however, some serious objections were raised regarding both of these papers. Cf. [**11**, pp. 408–414, 438], [**29**, pp. 731–741], and [**53**, pp. 233–236].

Partly for this reason, mathematicians sought to develop a more direct approach to the uniformization theorem. Such an approach was finally carried out by Koebe [**32**] and Poincaré [**56**] in 1907. Suffice it to say that the *direct* approach is based on potential theory, not on the continuity method. Some additional references are as follows: [**27**, pp. 73, 323], [**33**], [**52**], [**69**, pp. 159–179].

Although the continuity method ran into trouble (and was later abandoned), the underlying idea is still *very* tempting. Poincaré's approach [**53**] is of particular interest here, because of its connection with ordinary differential equations.

When viewed in its most primitive form, the inter-relationship between *conformal mapping and differential equations* certainly goes back to Riemann [**59**]. Compare: [**28**, pp. 214–216] and [**64**]. These techniques were first applied to uniformization problems around 1880, when Poincaré [**53**] showed that the differential equations characterizing uniformization depend upon $3g - 3$ complex parameters. Cf. equation (10) below. If these parameters could be determined, it would then be possible to compute the desired uniformization explicitly. Unfortunately, these parameters (known in the literature as *accessory parameters*) are notoriously difficult to get hold of. In fact, part of the difficulty with the original continuity method arose from trying to understand what happens to these parameters when the Riemann surface degenerates; see [**53**, §§8–14]. Cf. also [**6**], [**29**, pp. 731–741, 774], [**41**], and [**61**, pp. 215–304].

Since no one has succeeded in writing down explicit formulas for the accessory parameters, it seems perfectly reasonable to compromise and try to obtain variational formulas instead. The obvious hope is that such formulas will offer some insight into what is going on. Part 1 will be devoted to this problem.

At first glance, our results would seem to be rather disappointing; the variational equations are so messy that they appear to be useless. However, after staring at the formulas for a few minutes, one discovers some very

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