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BOOK REVIEWS

The wave equation on a curved space-time, by F. G. Friedlander, Cambridge Monographs on Mathematical Physics, no. 2, Cambridge Univ. Press, New York and London, 1976, x + 282 pp., \$39.50.

The geometrical theory of Cauchy's problem for linear hyperbolic partial differential equations with variable coefficients was originated by Hadamard and, years later, reshaped by Marcel Riesz. Now F. G. Friedlander in the book under review incorporates both Hadamard's and Riesz's ideas in a new formulation of the geometric theory in terms of distributions.

To discuss this subject, we begin with Hadamard. Summarizing his work on Cauchy's problem in a lecture series in 1920 [5], [6], Hadamard stressed the underlying analogy between elliptic and hyperbolic equations, and, through several major innovations, showed how Green's formula can be adapted from its original context of potential theory to Cauchy's problem for the wave and other hyperbolic equations. Both the analogy Hadamard referred to, and the obstacles to applying Green's formula to hyperbolic equations in the way he intended, are apparent when we compare, for instance, the three-dimensional Laplacian

$$\Delta_3 = \partial^2/\partial x_1^2 + \partial^2/\partial x_2^2 + \partial^2/\partial x_3^2$$

and the three dimensional d'Alembertian

$$\Box_3 = \partial^2/\partial x_2^2 - \partial^2/\partial x_2^2 - \partial^2/\partial x_3^2.$$

Let $\xi = (\xi_1, \xi_2, \xi_3)$ denote any fixed point, and $x = (x_1, x_2, x_3)$ a variable point, in real three-dimensional Euclidean space E^3. If $d(\xi, x)$ is the Euclidean distance $\sqrt{(x_1 - \xi_1)^2 + (x_2 - \xi_2)^2 + (x_3 - \xi_3)^2}$, and $r(\xi, x)$ the Lorentzian distance $\sqrt{(x_1 - \xi_1)^2 - (x_2 - \xi_2)^2 - (x_3 - \xi_3)^2}$, between x and ξ, then Laplace's equation $\Delta_3 u = 0$ has the solution $U_3(\xi, x) = 1/d(\xi, x)$ for $d(\xi, x) > 0$ and the homogeneous wave equation $\Box_3 v = 0$ the solution $V_3(\xi, x) = 1/r(\xi, x)$ for $r(\xi, x)$ real and > 0. The first function $U_3(\xi, x)$ enters the theory of Poisson's equation $\Delta_3 u = f$. In fact, if $u(x)$ is a solution of the equation on, say, a finite, smoothly bounded domain Ω, then a representation of $u(\xi)$, for any $\xi \in \Omega$, is obtained by using $U_3(\xi, x)$ as auxiliary function in Green's formula on the subregion of Ω on which $d(\xi, x) > \varepsilon$ and then letting $\varepsilon \downarrow 0$. Hadamard's theory made it possible to use $V_3(\xi, x)$ in an analogous procedure pertaining to the inhomogeneous wave equation $\Box_3 v = f$ say on the half-space $E^3_+ = \{x = (x_1, x_2, x_3): x_1 > 0\}$. The eventual outcome of that procedure is a representation of $v(\xi)$, for any $\xi \in E^3_+$, in terms of the values of v, $\partial v/\partial x_1$ and their derivatives with respect to x_2, x_3, on the initial surface $x_1 = 0$. The first step in Hadamard's method is to use $V_3(\xi, x)$ as auxiliary function in an integral formula analogous to Green's formula, but pertaining