BULLETIN OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 84, Number 1, January 1978

ON A PROBLEM OF ROTA

BY E. RODNEY CANFIELD

Communicated by J. R. Goldman, August 8, 1977

Let $S(n, k)$ denote the Stirling numbers of the second kind, and let K_n be such that $S(n, K_n) \geq S(n, k)$ for all k. Rota's problem [3] is to prove or disprove the following:

For all n, the largest possible incomparable collection of partitions of an n-set contains $S(n, K_n)$ partitions.

An "incomparable collection" of partitions is one in which no partition in the collection is a refinement of some other partition in the collection.

DEFINITION. Let $\mathcal{S}(n, k)$ denote the collection of all partitions of an n-set into k nonempty blocks. If $\mathcal{C} \subseteq \mathcal{S}(n, k)$, define $\mathrm{Span}(\mathcal{C})$ by

$$\mathrm{Span}(\mathcal{C}) = \{\pi \in \mathcal{S}(n, k + 1)\colon \pi \text{ is a refinement of some } \pi' \in \mathcal{C}\}.$$

THEOREM. *For all sufficiently large n, there is a collection $\mathcal{C} \subseteq \mathcal{S}(n, j)$ such that*

(i) $j + 1 = K_n$,

(ii) $|\mathrm{Span}(\mathcal{C})| < |\mathcal{C}|$, *where* $|\ |$ *denotes cardinality.*

Consequently, $(\mathcal{S}(n, j + 1) - \mathrm{Span}(\mathcal{C})) \cup \mathcal{C}$ *is an incomparable collection with more than* $S(n, K_n)$ *partitions.*

REMARKS. $\mathcal{C}$ *consists of all* $\pi \in \mathcal{S}(n, j)$ having exactly l blocks of size $\leq M$ and exactly $j - l$ blocks of size $> M$ and $\leq 2M$, where l and M are appropriately defined.

The proof of the Theorem requires [2] to estimate $|\mathcal{C}|$ and $|\mathrm{Span}(\mathcal{C})|$; and also requires [1] to know the approximate value of K_n.

REFERENCES

1. E. Rodney Canfield, *On the location of the maximum Stirling number(s) of the second kind*, 1977 (preprint).

2. ———, *Application of the Berry-Esséen inequality to combinatorial estimates*, 1977 (preprint).

3. G.-C. Rota, *Research problem* 2-1, J. Combinatorial Theory 2 (1967), 104.

DEPARTMENT OF STATISTICS AND COMPUTER SCIENCE, UNIVERSITY OF GEORGIA, ATHENS, GEORGIA 30602

AMS (MOS) subject classifications (1970). Primary 05A17, 05A05; Secondary 05C35, 06A10, 05C99.