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EXISTENCE AND APPLICATIONS OF REMOTE POINTS

BY ERIC K. VAN DOUWEN

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All spaces are completely regular, X^* is $\beta X - X$.

A point p of X^* will be called a *remote point of X* if $p \notin \mathrm{Cl}_{\beta X} D$ for every nowhere dense $D \subseteq X$. Fine and Gillman, [*FG*] showed that **Q**, the rationals, and **R**, the reals, have remote points if *CH* holds; their proof shows that X has remote points if X is separable and not pseudo-compact. We prove the existence of remote points without assuming additional set theoretic axioms, under slightly stronger conditions on X.

Recall that a *π-base* (or *pseudo-base*) for a space X is a family $\mathcal{B}$ of nonempty open sets such that every nonempty open set of X includes a member of $\mathcal{B}$. The *π-weight* of a space is the smallest cardinality for a π-base.

THEOREM A. *If X is a nonpseudocompact space with countable π-weight, then X has $2^{\mathfrak{c}}$ remote points.*

I originally proved this only for $X = \mathbf{Q}$, improving a technique from [vD_1]. I am indebted to Mary Ellen Rudin for showing me how to make my ideas work for $X = \mathbf{R}$. The above theorem is a further improvement.

For applications we need a "pointed" version of extremal disconnectedness.

DEFINITION. *If $p \in X$,* then X is called *extremally disconnected at p* if for all disjoint open $U, V \subseteq X$, $p \notin \overline{U} \cap \overline{V}$.

One can show that βX is extremally disconnected at every remote point of X. Without much effort one deduces the following theorem. (X is *nowhere locally compact* if no point has a compact neighborhood.)

THEOREM B. *Let X be a nonpseudocompact space with countable π-weight.*

(a) *βX is extremally disconnected at some point of X^*.*

(b) *If X is nowhere locally compact, X^* is extremally disconnected at some point.*

Frolík, [*F*], proved *that* X^* is not homogeneous if X is not pseudocompact. Theorem B can be used to show *why* X^* is not homogeneous, for suitable X.

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