BULLETIN OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 84, Number 1, January 1978

CONFIGURATION SPACES: APPLICATIONS TO GELFAND-FUKS COHOMOLOGY

BY F. R. COHEN AND L. R. TAYLOR[1]

Communicated by Philip Church, May 9, 1977

Let M be a manifold and define $F(M, k)$ as the subspace of M^k given by $\{(x_1, \ldots, x_k)|x_i \neq x_j \text{ if } i \neq j\}$. Permuting the coordinates gives a free action of Σ_k, the symmetric group on k letters on $F(M, k)$. If X is a based space, $X^{[k]} = X \wedge \cdots \wedge X$ supports a Σ_k action and we can form

$$B(M, X, k) = F(M, k) \times_{\Sigma_k} X^{[k]}/F(M, k) \times *.$$

The cohomologies of $F(M, k)$ and $B(M, X, k)$ have ubiquitous applications. $H^*(B(M, X, k))$ can be used to evaluate the E_2 term of a spectral sequence converging to the Gelfand-Fuks cohomology of M, [7] or [8]. It can also be used to evaluate the E_2 term of a spectral sequence due to P. Trauber [12] and D. W. Anderson [1] converging to the cohomology of the space of based maps from M to X. The calculations for the case $M = R^m$ give a complete and useful theory of homology operations for m-fold loop spaces [5].

In [4] and [5], the first author has obtained complete information on $H^*(F(R^m, k))$ and $H^*(B(R^m, X, k))$ in conjuction with his work on m-fold loop spaces. In this paper we give some calculations for some other manifolds M. We are most successful with $M^m = R^n \times V$ and with $M = S^m$.

Recall that by [4], $H^*F(R^m, k)$ is generated as an algebra by elements A_{ij} of degree $m - 1$ with $k \geqslant i > j \geqslant 1$ subject to the relations $A_{ir}A_{is} = A_{sr}(A_{is} - A_{ir})$ if $r \leqslant s$. With $A_{ji} = (-1)^m A_{ij}$ for $i > j$, the action of Σ_k is given by $\sigma^* A_{ij} = A_{\sigma i, \sigma j}$.

THEOREM 1. *If V is connected, if $M^m = R^n \times V$ with $n \geqslant 2$, and if all coefficients are in some field, $H^*(F(m, k))$ is isomorphic as an algebra to*

$$H^*(F(R^m, k)) \otimes H^*(V^k)/I$$

where I is the two-sided ideal generated by the elements

$$A_{ij} \otimes (1^{i-1} \times y \times 1^{k-i} - 1^{j-1} \times y \times 1^{k-j})$$

for all i and j and $y \in H^*(V)$. Both $H^*(F(R^m, k))$ and $H^*(V^k)$ are Σ_k algebras

AMS (MOS) subject classifications (1970). Primary 22E65, 55B99.
Key words and phrases. Configuration spaces, Gelfand-Fuks cohomology.
[1]Both authors are partially supported by an NSF grant MCS 76-07158 A01.