be used to refute this proposal; as in [4].) But on the borderline of game theory, in the subject of so-called Borel games, the use of uncountably many iterations of the power set operation has turned out to be demonstrably essential for solving problems about **R** (at least, if 'subsystems' of set theory are to be used, as in Martin's proof of the determinateness of all Borel games [Ann, of Math. (2) **102** (1975), 363–371]). Of course, this is a far cry from number theory and from those 'extravagantly' large cardinals which Gödel had in mind. (iii) To supplement the text where (a) spectacular uses of the skeptical, and (b) modest uses of the speculative tradition are given: (a′) in physics, the atomic theory is the standard example of a success fitting into the speculative tradition (atoms being hardly much more plausible than ghosts, from ordinary experience); in mathematics, nonconstructive methods. (b′) Modest uses abound of course; cf., for example, my review of Brouwer's work in **83** (1977) of this Bulletin (around p. 88).

Remark. It cannot have escaped the reader's notice that there is no counterpart in current foundations to what is surely the most glaring difference between modern natural science and the early speculations referred to in (i): the skillful use of a massive amount of empirical data. Certainly the history of mathematics–not, of course, mere snippets as in (i)–(iii) above–would seem to provide, at present, the most obvious source of empirical data for the general questions behind t.f., and, in particular, for a scientific study of Bourbaki's 'intuitive resonances' (in [1]). Of course, precautions are needed against overliteral interpretations of the data (cf. end of [2] about misplaced textual criticism) and artifacts (cf. *Remark* (ii) in [8]); as in all sciences, only more so because here the influence of the observer on the observation is particularly strong. The use of statistical data, as in [9], over long periods provides *one* way of taking precautions. It may well be that this historical perspective would be bad for mathematical practice (with busybodies drawing premature 'practical' conclusions from ill-digested data). But in the reviewer's opinion it is certainly good for foundational research, specifically, for *opening up this subject to* (genuine) *problems raised by recent computer-assisted proofs*: (a) Historically–and scientifically, if not artistically–speaking, such proofs, for example, of the 4-color conjecture, involve incomparably more progress than, say, the use of large cardinals in (ii) above. Compare the effort which would be needed to explain large cardinals to Archimedes with getting him to understand, let alone put together the largish computer used by Haken and Appel (and compare the general interest of the four color conjecture with that of Borel determinacy). (b) There are genuine doubts about the reliability of computer-aided proofs not resolved by the particular idealizations of reliability, that is, the doctrines of rigor in various branches of t.f. Inasmuch as reliability is a principal topic of foundations, these new proofs present novel data for foundations: it would seem premature (to put it mildly) to *assume* that these new data are less fundamental than the matters of 'principle' stressed in t.f.

G. KREISEL

BULLETIN OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 84, Number 1, January 1978

The theory of numbers, S. Iyanaga, ed., (translated by K. Iyanaga), North-Holland, Amsterdam; American Elsevier, New York, 1975, xi + 541 pp., \$51.95.

The Legendre symbol $(\frac{a}{p})$ is defined for any odd prime p and any rational integer a that is not divisible by p. It is equal to $+1$ or to -1 according as the congruence $x^2 \equiv a \bmod p$ does or does not have a solution in the ring of rational integers **Z**. The quadratic law of reciprocity then states that the equations

$$\left(\frac{p}{q}\right)\cdot\left(\frac{q}{p}\right) = (-1)^{(p-1)/2\cdot(q-1)/2}$$

and

$$\left(\frac{-1}{p}\right) = (-1)^{(p-1)/2}, \qquad \left(\frac{2}{p}\right) = (-1)^{(p^2-1)/8},$$