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VARIATIONAL INEQUALITIES AND FREE BOUNDARY PROBLEMS[1]

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The study of variational inequalities and free boundary problems finds application in a variety of disciplines including physics, engineering, and economics as well as potential theory and geometry. In this brief informal exposition, we intend to discuss several examples with the intention of illustrating the questions and ideas which comprise this theory. A general introduction to variational inequalities is provided in the papers **[Li-S]**, **[L-S1]**, and **[Br1]**. Free boundary problems from the viewpoint of variational inequalities are discussed in **[L-S1]**, **[K3]**, **[K4]**, and **[Ba1]**. We shall cite more recent work in the course of our discussion. The first four sections of this paper concern some familiar problems whereas the last three are devoted to very recent considerations in the study of free boundaries. Our understanding of this topic is still rudimentary.

The methods we discuss may be applied to treat problems of physical interest. Among these are questions of fluid mechanics (**[Br-S2]**, **[BR-D]**), hydraulics (**[Ba1]**, **[Ba2]**, **[S4]**, **[C1]**, **[F-J1]**, **[F-J2]**, **[Al]**), elasticity (**[Ti]**, **[Br2]**, **[Br-S1]**, **[C-N3]**), plasma physics (**[Te2]**, **[Be-Br]**, **[P]**, **[K-Sp]**), and hydrodynamics **[Fr-Ber]**. More complete references are given in the text.

Stopping time and impulse control problems lead to the notion of quasi-variational inequalities. Spatial limitations have prevented the inclusion of this topic here. We refer to **[Ben-Li1]**, **[Ben-Li2]**, **[F-Ben]**, **[F4]**, **[An-F]**. We have also omitted a discussion of the theory of thin obstacles (**[Fre2]**, **[Fre3]**, **[Giu]**).

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Contents

1. The obstacle problem. We begin with an informal description of what has been generally referred to as the "obstacle problem." Let $\Omega \subset \mathbf{R}^n$ be open, connected with smooth boundary $\partial\Omega$ and let $\psi \in C^2(\bar{\Omega})$ satisfy

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