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DETERMINATION OF THE AUGMENTATION TERMINAL FOR FINITE ABELIAN GROUPS

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Let G be a finite abelian group, and let IG denote the augmentation ideal in the integral group ring ZG. The graded ring associated with the filtration on ZG determined by the powers of IG is

$$\operatorname{gr} ZG = \sum_{n \geqslant 0} \oplus IG^n / IG^{n+1}.$$

We write $Q_n G = IG^n / IG^{n+1}$. As is well known [1], [6], the sequence $Q_n G$ becomes stationary after a finite number of steps. We call its terminal value the *augmentation terminal*, $Q_\infty G$. We outline here a method for investigating $Q_\infty G$ for any G.

An obvious splitting allows us to assume that G is a p-group.

We choose a generator for each cyclic direct factor of G. Let Γ be our set of such generators, and let $\Lambda = \{\lambda | \lambda + 1 \in \Gamma\}$. Generalizing Lemma 2 of [3] we have

LEMMA. *For $n \geqslant 1$ the set of n-fold products of elements of Λ generates IG^n; a fortiori it generates $Q_n G$.*

If $\lambda \in \Lambda$ *there is an integer r such that* $(\lambda + 1)^{p^r} - 1 = 0$. Furthermore, by the structure of G, these equations are the only possible source of relations among the elements of Λ. Hence we have immediately

THEOREM 1. *Let $f(\lambda_1, \ldots, \lambda_k)$ be a nontrivial relator in $Q_n G$, where the $\lambda_i \in \Lambda$. Let $\lambda_i + 1$ be of order p^{r_i} in G, each i. Let $X_1, \ldots, X_k$ be indeterminates over $\mathbf{Z}$. Then there are polynomials $h_i(X_1, \ldots, X_k)$ with integer coefficients such that*

$$f(X_1, \ldots, X_k) - \sum_{i=1} \{(X_i + 1)^{p^{r_i}} - 1\} h_i(X_1, \ldots, X_k)$$

has no terms of degree $\leqslant n + 1$.

Actually using this result to find relators is far from easy, as the references show [2], [7], [8]. If G is an elementary p-group we have

THEOREM 2. *The relators in $Q_\infty G$ are generated by $\{p\lambda | \lambda \in \Lambda\}$ and $\{\lambda^p \mu - \lambda \mu^p | \lambda, \mu \in \Lambda\}$.*

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