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QUASICONFORMAL MAPPINGS, WITH APPLICATIONS TO DIFFERENTIAL EQUATIONS, FUNCTION THEORY AND TOPOLOGY

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The theory of quasiconformal mappings is nearly 50 years old (see **[44]** for references to the papers by Grötzsch, Ahlfors Lavrent'ev and Morrey from the 20's and 30's) and the interest in them does not seem to wane. These mappings may be studied for their own sake or as a tool for attacking other mathematical problems; they are indeed a powerful and flexible tool. The purpose of this lecture is to state two basic theorems about quasiconformal mappings in two dimensions (the existence theorem, the first version of which is due to Gauss, and Teichmüller's theorem about extremal quasiconformal mappings) and to discuss some applications of these theorems. The discussion will necessarily be sketchy and several important topics will be slighted or not even mentioned. (Some of those are covered in **[15]**, **[20]**, **[23]**.)

(At St. Louis I learned about an interesting application, due to J. Sachs and K. Uhlenbeck, to the theory of minimal surfaces. Added in proof.)

Lack of time and of competence prevents me from saying anything about the subtle theory of quasiconformal mappings in n-space, initiated in a short note **[48]** by my late teacher Loewner, and developed by Gehring, Väisälä and others (see the references in **[32]**, **[67]**), a theory which also has important applications (Mostow **[55]**).

We begin by defining the concept of quasiconformality.

1. Recall that a Riemannian metric in a domain in the (x,y) plane is defined by a quadratic differential form

$$ds^2 = E(x,y)\,dx^2 + 2F(x,y)\,dx\,dy + G(x,y)\,dy^2 \tag{1}$$

with

$$EG - F^2 > 0, \qquad E > 0. \tag{2}$$

Setting $x + iy = z$, $x - iy = \bar{z}$, (1) can be rewritten as

$$ds^2 = \Lambda(z)^2|dz + \mu(z)\,d\bar{z}|^2 \tag{1'}$$

where Λ is a real-valued and μ a complex-valued function. Condition (2) becomes

$$\Lambda > 0, \qquad |\mu| < 1. \tag{2'}$$

A mapping $x + iy = z \mapsto w = u + iv$ is called conformal with respect to ds if it preserves orientation and, except at isolated points, takes angles measured by the metric (1) into equal Euclidean angles.

Using the standard notations

$$\frac{\partial}{\partial z} = \frac{1}{2}\left(\frac{\partial}{\partial x} - i\frac{\partial}{\partial y}\right), \qquad \frac{\partial}{\partial \bar{z}} = \frac{1}{2}\left(\frac{\partial}{\partial x} + i\frac{\partial}{\partial y}\right),$$

the requirement that w be conformal with respect to the metric (1) may be

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