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AMENABLE ERGODIC ACTIONS, HYPERFINITE FACTORS, AND POINCARÉ FLOWS[1]

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1. Introduction. In this paper we announce the introduction of a new notion of amenability for ergodic group actions and ergodic equivalence relations. Amenable ergodic actions occupy a position in ergodic theory parallel to that of amenable groups in group theory and one can therefore expect this notion to be useful in diverse circumstances. Here we announce applications to hyperfinite factor von Neumann algebras, skew products, Poincaré flows, and Poisson boundaries of random walks. We remark that from Mackey's virtual group viewpoint, we are considering amenable virtual subgroups of locally compact groups.

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Full details and related results will appear elsewhere.

2. Preliminaries. A group is amenable if every continuous affine action (i.e., homomorphism into the affine group) on a compact convex set has a fixed point. There is a strong parallel between homomorphisms in group theory and cocycles in ergodic theory [2], [3], [6], [7]. The condition of amenability for ergodic actions which we now spell out is essentially the condition that every cocycle into an affine group has a fixed element in some appropriate sense.

Let G be a locally compact second countable group, S a standard Borel space with a Borel right G-action, and m a probability measure on S quasi-invariant and ergodic under G. Let E be a separable Banach space, $\mathrm{Iso}(E)$ the group of isometric isomorphisms with the strong operator topology, and E_1^* the unit ball in the dual of E with the $\sigma(E^*, E)$ topology. Suppose $c: S \times G \longrightarrow \mathrm{Iso}(E)$ is a (Borel) cocycle, i.e., for all $g, h \in G$, $c(s, gh) = c(s, g)c(sg, h)$ a.e. Let c^* be the induced adjoint cocycle, i.e., $c^*(s, g) = (c(s, g)^{-1})^*: E^* \longrightarrow E^*$. Suppose $s \longrightarrow A(s)$ is a Borel field of compact convex subsets of E_1^*, i.e., $\{(s, x) | x \in A(s)\} \subset S \times E_1^*$ is Borel. We call $A(s)$ c-invariant if for each g, $c^*(s, g)A(sg) = A(s)$ for almost all s.

DEFINITION 2.1. S is called an amenable G-space if for all E, c, and

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