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TIGHT AND TAUT EMBEDDINGS AND PROJECTIVE TRANSFORMATION

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Let $f: M^n \longrightarrow R^N$ be an embedding of an n-dimensional compact differentiable manifold M into a euclidean vector space R^N. Any hyperplane in R^N has an equation $z = c$, where z is a linear function and c a real number. For the oriented hyperplane $z = c$ we use the notation ($zf = z \circ f$ is the composition)

$$(zf)_c^- = \{x \in M: zf(x) < c\}$$

to indicate the image of f on the negative side of the hyperplane. The closure of $(zf)_c^-$ is

$$(zf)_c = \{x \in M: zf(x) \leqslant c\}.$$

The embedding f is called *tight* if for almost all z and c the inclusion $i:(zf)_c \longrightarrow M$ induces a monomorphism $i_*: H_*((zf)_c, F) \longrightarrow H_*(M, F)$ of homology groups, where the coefficient F is chosen to satisfy condition 3A of [5] (or (3) of [6]).

Besides the collection of hyperplanes, other interesting geometric objects are hyperspheres. A hypersphere of R^N is given by an equation $d_p(x) = c$, where $d_p(x)$ is the distance from x to a fixed point p of R^N and c is a positive real number. The hypersphere is naturally oriented so that its inner part is the positive side of the hypersphere. We define analogously the sets $(d_pf)_c^-$ and $(d_pf)_c$. The embedding f is called *taut* if for almost all p and $c > 0$ the inclusion i: $(d_pf)_c \longrightarrow M$ induces a monomorphism $i_*: H_*((d_pf)_c) \longrightarrow H_*(M)$ where the coefficient is chosen to be the same as before and hence is omitted. Originally both concepts, tight and taut, are defined in terms of the Morse height functions and Morse distance functions respectively, and this is why in both definitions we use "almost all". It seems that the definition with "almost all" should imply the one without "almost all". The answer for this is still unknown. However, for the special class of embeddings we are interested in, the definition with "almost all" is equivalent to the one without "almost all".

The geometrical content of an embedding $f: M^n \longrightarrow R^N$ is given by a symmetric tensor T defined as follows: For any two tangent vectors x, y of M at $p \in M$ we first extend x, y to vector fields X, Y respectively near p. We define

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