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UNIQUE CONTINUATION THEOREMS FOR SOLUTIONS OF PARTIAL DIFFERENTIAL EQUATIONS

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In this note we present a new general unique continuation theorem for solutions of linear P.D.E.'s with analytic coefficients. The solutions are assumed to vanish of infinite order on manifolds of codimension $\geqslant 1$. Holmgren's theorem and the unique continuation property of solutions of elliptic equations are particular cases of this theorem. Some new unique continuation results are also given for certain hyperbolic equations (and inequalities) with nonanalytic coefficients.

Let Ω be an open set of $\mathbf{R}^n$ and $P(x, D)$ be a linear partial differential operator of order m, with analytic coefficients in Ω. We denote by p_m the principal symbol of P and by Σ the characteristic set of P contained in $T^*(\Omega)\backslash 0$, i.e.

$$\sum = \{(x, \xi); x \in \Omega, \xi \in \mathbf{R}^n\backslash\{0\}, p_m(x, \xi) = 0\}.$$

If M and N are two differentiable manifolds contained in Ω, $M \subset N$, and if u is a continuous function defined in N, we say that *u vanishes of infinite order* on M if, for all $\alpha \in \mathbf{R}$, the function $x \longrightarrow d(x, M)^\alpha u(x)$ is bounded in any compact set of N. Here $d(x, M)$ denotes the distance of x from M.

We say that the manifold M is *P-noncharacteristic* if the normal bundle of M (in $T^*(\Omega)\backslash 0$) does not intersect Σ, i.e., for all $x \in M$ and $\xi \in \mathbf{R}^n\backslash\{0\}$ normal to M at x, $p_m(x, \xi) \neq 0$. We have:

THEOREM 1. *Let M and N be two analytic manifolds in Ω, $M \subset N$, and assume that M is P-noncharacteristic. There is a neighborhood V of M in N, such that if u is a continuous function in Ω satisfying*: (i) $Pu = 0$ *in Ω, and* (ii) *the restriction of u to N vanishes of infinite order on M, then u must vanish in V.*

Note that N can be taken to be Ω. Condition (ii) may be replaced by a weaker condition; for example, if M divides N into two sides, then it is enough

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