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OBSTRUCTION THEORY IN 3-DIMENSIONAL TOPOLOGY: CLASSIFICATION THEOREMS

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We consider the classification up to homotopy of homotopy equivalences of compact 3-manifolds. Given two compact 3-manifolds (with base point and CW-decomposition) (M, m) and (N, n), in [3] we found an algebraic criterion for the existence of a degree 1 map f: $(M, \partial M) \longrightarrow (N, \partial N)$ extending a given map f^1 defined on the relative 1-skeleton $(M, \partial M)^1$. Here we consider the space $\mathcal{H}^1(M, m)$ of degree 1 homotopy equivalences f: $(M, m) \longrightarrow (M, m)$ such that $f|\partial M = \mathrm{Id}$ and f is homotopic rel $\partial M \cup \{m\}$ to a map coinciding with the identity on $(M, \partial M)^1$. If $\partial M = \emptyset$, it is equivalent to say that f induces the identity automorphism of $\pi_1(M, m)$. (If $\partial M \neq \emptyset$, we assume that $m \in \partial M$.) Important results are the following.

1. Following Waldhausen e.a. [6] a homotopy equivalence of $\mathbf{P}^2$-irreducible (closed) sufficiently large 3-manifolds is homotopic to a homeomorphism unique up to isotopy. Our result indicates that the exclusion of 2-sided projective planes is necessary. Indeed, suppose M is the connected sum of two nonsimply connected 3-manifolds, then we have

THEOREM ([2]). *If M contains* 2*-sided projective planes, M admits a self homotopy equivalence, in $\mathcal{H}^1(M, m)$, which is not homotopic to a homeomorphism* rel ∂M.

Recall that all elements of $\mathcal{H}^1(M, m)$ are *simple* homotopy equivalences (in the sense of Whitehead).

On the other hand, let S be an embedded 2-sphere in M with collar $S \times [0, 1]$. Then the *rotation along S* is the homeomorphism in $\mathcal{H}^1(M, m)$ defined by the identity outside $S \times [0, 1]$ and by a generator of $\pi_1 SO(3)$ within $S \times [0, 1]$.

THEOREM ([4]). *Let M be a* 3*-manifold which does not contain* 2*-sided projective planes, then every self homotopy equivalence in $\mathcal{H}^1(M, m)$ is homotopic* rel $\partial M \cup \{m\}$ *to a rotation along a sphere.*

2. Let $\mathcal{R}$ be the set of 2-spheres S in M such that we can express $M = M_1 \cup M_2$, where $M_1 \cap M_2 = S$, and where $M_1 \cup_S D^3$ is a connected sum of closed manifolds, each either with finite fundamental group whose 2-Sylow

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