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THE GENERALIZED GAMMA FUNCTION, NEW HARDY SPACES, AND REPRESENTATIONS OF HOLOMORPHIC TYPE FOR THE CONFORMAL GROUP[1]

BY KENNETH I. GROSS, WAYNE J. HOLMAN, III AND RAY A. KUNZE

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Operator valued generalizations of the integral formula for the classical gamma function arise in connection with noncompact semisimple, or reductive, Lie groups for which the symmetric space G/K is Hermitian, and they relate to various problems in analysis, group representations, and number theory. In particular, when the holomorphic discrete series for G, constructed originally by Harish-Chandra [3], is realized in terms of the unbounded form of G/K as a Siegel domain, the gamma function plays a decisive role (cf., [1], [2a], [2b], [2d], [6a], [6b]). Moreover, the holomorphic discrete series has an analytic continuation [7], the full extent of which is controlled by the analytic continuation of a normalized version of the gamma function. In general, however, it is only when the gamma function is scalar valued, an occurrence which accounts for but a small part of the holomorphic discrete series, that the full analytic continuation has been determined. In that specialized context, it is known from [6b] that Hardy type Hilbert spaces associated to the various boundary components of G/K appear at the "integer points" in the analytic continuation.

This note announces rather complete solutions to these problems for the conformal group $G = U(2, 2)$. Specifically, we give the entire analytic continuation of the gamma function, the full extent of analytic continuation of the holomorphic discrete series, and we introduce some new vector-valued Hardy spaces.

I. The generalized gamma function. Let $\mathcal{A} = A \times A$ where $A = \mathrm{GL}(2, \mathbf{C})$ and fix a complete set of irreducible holomorphic finite-dimensional representations λ of $\mathcal{A}$ such that $\lambda(a_1, a_2)^* = \lambda(a_1^*, a_2^*)$. Let λ be parametrized by a pair of highest weights $(\sigma_j + 2l_j, \sigma_j)$, $(j = 1, 2)$, where σ_j and $2l_j$ are integers and $l_j \geqslant 0$. Then $\lambda = \lambda(\,\cdot\,; \sigma_1, \sigma_2, \lambda^0)$ where

$$\lambda(a_1, a_2) = \Delta(a_1)^{\sigma_1}\Delta(a_2)^{\sigma_2}\lambda^0(a_1, a_2) \tag{1}$$

with $\Delta = \det$ and $\lambda^0 = \lambda^0(\,\cdot\,; l_1, l_2)$ a polynomial representation. Let P be

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