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SPACES OF SMOOTH FUNCTIONS ON ANALYTIC SETS

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1. **Stability.** Let $X \supset Y$ be real analytic (or, more generally, closed semianalytic) subsets of R^n with dim $X < n$, and let $M \subset N$ be submodules of $(C^\infty(R^n))^m$ obtained (as modules of global sections) on tensoring by $C^\infty(R^n)$ coherent real analytic subsheaves $\widetilde{M} \subset \widetilde{N}$ of $(\mathcal{O}(R^n))^m$, where $\mathcal{O}(R^n)$ denotes the sheaf of real analytic functions on R^n. Let $M(Y, X)$ (similarly for N) be the space of m-tuples ϕ of Taylor fields on X flat on Y such that at each point $x \in X$, ϕ_x is in the formal completion M_x of M at x. Let r: $N(Y, R^n) \longrightarrow N(Y, X)/M(Y, X) = P(Y, X)$ denote the restriction.

THEOREM 1. *There is a continuous E: $P(Y, X) \longrightarrow N(Y, R^n)$ such that $rE = 1$.*

Theorem 1 is proved using the approach of [1, Chapter 6], where it is shown that r: $N(Y, R^n) \longrightarrow N(Y, X)$ is onto, with modifications as in [5]; E is nonlinear.

The ideal I of analytic functions vanishing on a real analytic set need not be coherent, but using a suitable decomposition of X by (nonclosed) semianalytic subsets, on each of which I is globally generated, Theorem 1 can be applied to give, with $E(Y, X)$ denoting the space of smooth functions on X flat on Y.

THEOREM 2. *There is a continuous E: $E(Y, X) \longrightarrow E(Y, R^n)$, a right inverse for the restriction.*

J. Mather's proof ([2, in particular, p. 283 and following]), can then be applied to give

COROLLARY 1. *Infinitesimal stability implies stability for smooth proper mappings of X into a manifold.*

2. ***G*-manifolds.** Let G be a compact Lie group acting linearly on R^n and let ϕ: $R^n \longrightarrow R^m$ be a polynomial "Hilbert" map (i.e. ϕ induces a mapping from the polynomials on R^m onto the G invariant polynomials on R^n). Let $X \subseteq R^n$ be a G invariant analytic set and let $C_G^\infty(X)$ denote the space of G invariant smooth functions on X. The method of Theorem 1 (see [5]) gives

THEOREM 3. *There is a continuous E: $C_G^\infty(X) \longrightarrow C^\infty(R^m)$ such that $\phi^*E = 1$.*

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