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SINGULAR INVARIANT EIGENDISTRIBUTIONS AS CHARACTERS

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1. Let G be a connected, acceptable linear real semisimple Lie group with finite center, and let K be a maximal compact subgroup of G. We assume that $\operatorname{rank}(K) = \operatorname{rank}(G)$, and we let T be a Cartan subgroup of G contained in K. We denote by $\mathfrak{G}_C$ and $\mathfrak{t}_C$ the complexifications of the Lie algebras of G and T respectively. The character group of T may be identified with a lattice L_T in the dual of $\sqrt{-1}\mathfrak{t}$, and the Weyl group W_C of the pair $(\mathfrak{G}_C, \mathfrak{t}_C)$ acts on L_T. We say that $\lambda \in L_T$ is *regular* if $w\lambda \neq \lambda$ for all $w \neq 1$ in W_C; otherwise λ is said to be *singular*. We denote the set of singular λ by L_T^s.

To each $\lambda \in L_T$, Harish-Chandra has associated a tempered invariant eigendistribution $\Theta(\lambda)$ on G ([1], [2]), and, if λ is regular, $\Theta(\lambda)$ is (up to a sign) a discrete series character of G. Our interest is centered on the distributions $\Theta(\lambda)$, λ singular, which we call *singular invariant eigendistributions associated to* T. More generally, we consider a class of singular invariant eigendistributions associated to any conjugacy class of Cartan subgroups of G.

The singular invariant eigendistributions mentioned above appear in the explicit formula for the Fourier transform of certain orbital integrals on G (see [3], [8]). The goal of this note is the character theoretic identification of these singular distributions. We first use a result of Zuckerman [12] which states that the tempered invariant eigendistributions on G which are "limits of discrete series" are actually characters on G. Then, we embed these characters in unitary principal series representations of G by appealing to a theorem of Hirai [5] which, for a restricted class of real simple Lie groups, characterizes those tempered invariant eigendistributions which are uniquely determined by their restriction to a distinguished Cartan subgroup in their support. In a recent note [4], the first author has removed the restrictions on G and has proved Hirai's theorem for any connected, acceptable, reductive Lie group with compact center.

In the case when G has split rank equal to one, the results announced in this note were worked out in part several years ago with K. Okamoto, and, more recently, in complete detail with N. Wallach. We note that our work overlaps with the recent work of Schmid [9] and Knapp and Zuckerman [7], but both our motivation and our techniques of proof are different.

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