BULLETIN OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 83, Number 1, January 1977

# A STRUCTURAL GENERALIZATION OF THE RAMSEY THEOREM

BY JAROSLAV NEŠETŘIL AND VOJTĚCH RÖDL

Communicated by Solomon Feferman, August 30, 1976

ABSTRACT. A generalization of the Ramsey theorem is stated. This solves a problem of P. Erdős and others. The result has recent applications in the theory of ultrafilters and model theory.

The Ramsey theorem [3] states:

*For all positive integers* $k, m, p$ *there exists an* $n$ *such that for every coloring* $c\colon [n]^p \longrightarrow k$, *there exists a homogeneous* $m$ *set*, $M \subseteq n$, $|M| = m$, *with* $|c([M]^p)| = 1$.

This can be generalized to set systems of a given type and to set systems without forbidden subsystems. The purpose of this note is to announce this result.

A family $\Delta = (\delta_i; i \in I)$, $\delta_i \geqslant 1$, is called a type. $(X, \mathcal{M}) = (X, (\mathcal{M}_i; i \in I))$ is a set system of type $\Delta$ if $\mathcal{M}_i \subseteq [X]^{\delta_i}$ and $X$ is a finite ordered set. $f\colon (X, \mathcal{M}) \longrightarrow (Y, \mathcal{N}) = (Y, (\mathcal{N}_i; i \in I))$ is called an embedding if $f\colon X \longrightarrow Y$ is a monotone 1-1 mapping and $f(M) \in \mathcal{N}_i \Longleftrightarrow M \in \mathcal{M}_i$ for every $i \in I$. $(X, \mathcal{M})$ is a subsystem of $(Y, \mathcal{N})$ if the inclusion $X \subseteq Y$ is an embedding. Denote by $\mathrm{Emb}(A, B)$ the set of all embeddings $A \longrightarrow B$ and by $\mathrm{Set}(\Delta)$ the category of all set systems of type $\Delta$ and all embeddings.

The following holds:

THEOREM. *Let a type* $\Delta$ *be fixed. Let* $k$ *be a positive integer and* $A \in \mathrm{Set}(\Delta)$. *Then for every* $B \in \mathrm{Set}(\Delta)$ *there exists* $C \in \mathrm{Set}(\Delta)$ *such that the following holds: for every coloring* $c\colon \mathrm{Emb}(A, C) \longrightarrow k$ *there exists a subsystem* $B'$ *of* $C$ *which is isomorphic to* $B$ *such that* $|c(\mathrm{Emb}(A, B'))| = 1$. *Moreover, if* $B$ *does not contain a fundamental set system* $D$, *then* $C$ *may be chosen with the same property. Here* $D = (X, \mathcal{M})$ *is fundamental if for every* $i \in I$ *either* $\mathcal{M}_i = \emptyset$ *or* $\mathcal{M}_i = [X]^{\delta_i}$.

This generalizes the Ramsey theorem and has the following consequences:

COROLLARY 1. *For every graph* $G = (V, E)$ *without a complete graph with k-vertices, there exists a graph* $H = (W, F)$ *without a complete subgraph*

---

*AMS (MOS) subject classifications* (1970). Primary 05A99; Secondary 04A20, 02H05.
*Key words and phrases.* Ramsey theorem, set systems, partition.