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THE FUNCTIONS OPERATING ON CERTAIN ALGEBRAS OF MULTIPLIERS[1]

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In this note, we announce a new result concerning functions operating on multiplier algebras. We begin by introducing the following notation. Let G be a LCA group with dual group Γ. $M(G)$ will denote the algebra of finite, regular Borel measures on G. Let $M_0(G) = \{\mu \in M(G) | \hat{\mu}$ vanishes at ∞ on $\Gamma\}$. If $1 \leqslant p < \infty$, let $M_p(G)$ denote the class of multiplier transformations on $L_p(G)$. If $T \in M_p(G)$, $\hat{T}$ will be the unique function in $L_\infty(\Gamma)$ so that $T(f)\hat{} = \hat{T}\hat{f}$, for all integrable simple functions f. Finally, we write $C_0M_p(G) = \{T \in M_p(G) | \hat{T}$ is continuous and vanishes at ∞ on $\Gamma\}$.

Suppose that G is nondiscrete. It is well known that only entire functions operate on the Banach algebra $M(G)$ [3, Chapter 6]. This result was strengthened in [1]. There, Igari showed that only entire functions operate from $M(G)$ into the algebra $M_p(G)$, $1 < p < \infty, p \neq 2$. In [4], Varopoulos showed that for compact G, only entire functions operate on $M_0(G)$. We have the following theorems, which, in a sense, may be viewed as the L_p analogues of the aforementioned result of Varopoulos.

THEOREM 1. *Let $1 < p < \infty$ with $p \neq 2$. Suppose that F: $[-1, 1] \longrightarrow \mathbf{C}$ and that F operates on the algebra $C_0M_p(\mathbf{T}^n)$. Then F coincides with an entire function in some neighborhood of* 0.

THEOREM 2. *Let $1 < p < \infty$ with $p \neq 2$, and let G denote one of the groups $\mathbf{R}^n$ or $\mathbf{Z}^n$. Suppose that F: $[-1, 1] \longrightarrow \mathbf{C}$ and that F operates on the algebra $C_0M_p(G)$. Then F coincides with an entire function on* $[-1, 1]$.

These results complete the investigation begun by the author in [5]. We now indicate some of the ideas involved in the proof.

Assume that $G = \mathbf{T}$ and $1 < p < 2$. By standard arguments (see [1] and [3, Chapter 6]) we may assume that $F(x) = \Sigma_{k=1}^\infty a_k x^k$ for $|x| < \epsilon$. It then suffices to show that there exists j_ϵ such that

$$|a_j| \leqslant C_\epsilon 10^j \tag{1}$$

for all $j \geqslant j_\epsilon$. This is accomplished by studying refinements of the multipliers considered in [5]. Corresponding to the sequence $\{a_j\}$, we construct measures $\{\lambda_j\}$, λ in $M(\mathbf{T})$ so that for all j

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