

BOOK REVIEWS

The large scale structure of space-time, by S. W. Hawking and G. F. R. Ellis, Cambridge Monographs on Mathematical Physics, Cambridge University Press, New York, 1975, 391 + xi pp., \$10.95 (paper), \$28.50 (cloth).

This is an exciting and important volume since it is the first comprehensive presentation of a theory of cosmology taking into account the discoveries of the past quarter century in particle physics, radio astronomy, and differential topology. The astronomical universe or cosmos is examined within the framework of general relativity and global differential geometry. The exposition is authoritative and painstaking, although in the search for logical completeness sometimes a bewildering tangle of alternatives and complexities is introduced (see, for instance, Chapter 6 on causal structure). The authors assume a basic knowledge of the physical aspects of general relativity theory, and write for the reader who is skilled in tensor calculus but who wishes to see the appropriate concepts defined in an intrinsic coordinate-free manner suitable for a global geometry.

Concerning the central thesis of their treatise, the authors write in the preface:

"The subject of this book is the structure of space-time on length scales from 10^{-13} cm, the radius of an elementary particle, up to 10^{28} cm, the radius of the universe. For reasons explained in Chapters 1 and 3, we base our treatment on Einstein's General Theory of Relativity. This theory leads to two remarkable predictions about the universe: first, that the final fate of massive stars is to collapse behind an event horizon to form a 'black hole' which will contain a singularity; and secondly, that there is a singularity in our past which constitutes, in some sense, a beginning to the universe. Our discussion is principally aimed at developing these two results. They depend primarily on two areas of study: first, the theory of the behaviour of families of timelike and null curves in space-time, and secondly, the study of the nature of the various causal relations in any space-time."

Thus, for the authors, a mathematical model of the space-time universe consists of

(i) A differentiable 4-manifold \mathfrak{M} (connected, Hausdorff, paracompact, C^∞ -manifold without boundary)—this represents the amorphous qualitative structure of the cosmos.

(ii) A Lorentz metric tensor \mathbf{g} , with components $g_{ab}(x)$ in any local chart (x^a) , $a = 1, 2, 3, 4$, of \mathfrak{M} (a symmetric covariant 2-tensor field of class C^r , $r \geq 2$, with the relativistic signature 2, or $(+++ -)$)—this represents the special relativistic or Minkowski structure on each tangent space T_p of \mathfrak{M} , and permits the construction of spacelike and nonspacelike (timelike and lightlike or null) tangent vectors in T_p . The nonspacelike vectors fill the lightcones in each T_p and these define the basic causal structure on \mathfrak{M} . Timelike and null geodesics define the world-trajectories or histories of free particles and light rays.