BULLETIN OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 82, Number 5, September 1976

ON MAXIMAL FINITE IRREDUCIBLE SUBGROUPS OF GL(n, Z)
I. THE FIVE AND SEVEN DIMENSIONAL CASE
II. THE SIX DIMENSIONAL CASE

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Communicated by Olga Taussky Todd, February 16, 1976

By the Jordan-Zassenhaus Theorem there is only a finite number of conjugate classes (called **Z**-classes) of finite subgroups of GL(n, **Z**). After various authors have determined all of these groups for $n \leqslant 4$ [4], [3], as well as the maximal finite subgroups of GL(5, **Z**) [2], [7], [8], we develop new methods for the determination of the absolutely irreducible maximal finite subgroups of GL(n, **Z**) and compute these groups for $n = 5, 6, 7$. (We remark that irreducibility is tantamount to absolute irreducibility in case n is an odd prime number.) The algorithm proceeds in three steps.

1. Every absolutely irreducible finite subgroup G of GL(n, **Z**) fixes, up to scalar multiples, exactly one positive definite symmetric matrix $X \in \mathbf{Z}^{n\times n}$ called the form of G:

$$g^T X g = X \quad \text{for all } g \in G.$$

It follows that each maximal finite absolutely irreducible subgroup H of GL(n, **Z**) is the full **Z**-automorph of its form. (The **Z**-automorph of a positive form is certainly finite.) But the form of H is already determined by each of the absolutely irreducible subgroups of H. So at step 1 we determine all finite minimal absolutely irreducible subgroups of GL(n, **Z**) up to conjugacy under GL(n, **Q**), i.e. those absolutely irreducible groups which do not contain any proper absolutely irreducible subgroups. This is essentially a task of classical representation theory. As for the primitive groups we refer to papers by Brauer [1], Wales [9], and Lindsey [5]. To find the imprimitive groups we first had to prove an integral version of Clifford's Theorem. For $n = 5$ and 7 there are 2 minimal absolutely irreducible groups to be considered, but 33 for $n = 6$ because 6 is no prime so that many imprimitive groups turn up.

2. Step 2 consists of finding the **Z**-classes of the groups determined at step 1 which was done by means of electronic computation using the centering algorithm developed in [6]. Let us describe the algorithm in module theoretic terms. Let L and M be **Q**-equivalent **Z**G-representation modules, i.e. $\mathbf{Q}L \cong_{\mathbf{Q}G} \mathbf{Q}M$, then M is **Z**-equivalent to a submodule M' of L of finite index in L. One can

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AMS (*MOS*) *subject classifications* (1970). Primary 20C10.
Key words and phrases. Integral matrix groups.