BULLETIN OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 82, Number 5, September 1976

DIHEDRAL SINGULARITIES: INVARIANTS, EQUATIONS AND INFINITESIMAL DEFORMATIONS

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Communicated by Robert Fossum, May 7, 1976

In this note we give a short survey on joint work with K. Behnke; details will appear in [1] and [2].

Let n, q be positive integers with $2 \leqslant q < n$ and $\gcd(n, q) = 1$, $m = n - q$. We define elements $\phi_m, \psi_q, \eta \in \mathrm{GL}(2, \mathbf{C})$ by

$$\phi_m = \begin{pmatrix} \zeta_{2m} & 0 \\ 0 & \zeta_{2m} \end{pmatrix}, \quad \psi_q = \begin{pmatrix} \zeta_{2q} & 0 \\ 0 & \zeta_{2q}^{-1} \end{pmatrix}, \quad \eta = \begin{pmatrix} 0 & i \\ i & 0 \end{pmatrix},$$

where $i = \sqrt{-1}$ and $\zeta_k = \exp(2\pi i/k)$. The group $G_{n,q} \subset \mathrm{GL}(2, \mathbf{C})$ is generated by

(a) ϕ_m, ψ_q, η in case m odd,

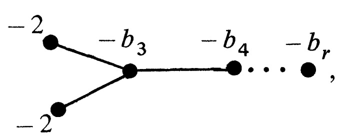
(b) $\psi_q, \eta \circ \phi_{2m}$ in case m even.

$G_{n,q}$ has finite order $4mq$; $G_{q+1,q}$ is the *binary dihedral group* of order $4q$.

$G_{n,q}$ acts on $\mathbf{C}^2$ in the usual way; the quotient $\mathbf{C}^2/G_{n,q}$ has precisely one (normal) complex-analytic singularity. We call it the *dihedral singularity of type* $D_{n,q}$. If we expand n/q into the modified continued fraction à la Hirzebruch-Jung,

$$n/q = b_3 - 1 \rfloor \overline{b_4} - \cdots - 1 \rfloor \overline{b_r}, \qquad b_\rho \geqslant 2, \; r \geqslant 4,$$

it can be characterized by the dual graph of its minimal resolution (cf. [3]):



$, \quad \bullet \cong \mathbf{P}_1(\mathbf{C}).$

The equations are calculated by invariant theory. As in the cyclic group case [5], we put

$$n/m = a_2 - 1 \rfloor \overline{a_3} - \cdots - 1 \rfloor \overline{a_{e-1}}, \qquad a_\epsilon \geqslant 2.$$

Further set $A_3 = a_3 + 1$, $A_\epsilon = a_\epsilon$, $\epsilon \neq 3$, and

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AMS (MOS) subject classifications (1970). Primary 32C40, 32G05, 14B05, 14J15; Secondary 32C45, 14E15.