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ON THE TOPOLOGICAL STRUCTURE OF SIMPLY-CONNECTED ALGEBRAIC SURFACES

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Suppose X is a smooth *simply-connected* compact 4-manifold. Let $P = CP^2$ and $Q = -CP^2$ be the complex projective plane with orientation opposite to the usual. We shall say that X is completely decomposable if there exist integers a, b such that X is diffeomorphic to $aP \# bQ$.

By a result of Wall [**W1**] there always exists an integer k such that $X \# (k + 1)P \# kQ$ is completely decomposable. If $X \# P$ is completely decomposable we shall say that X is almost completely decomposable. In [**MM1**] we demonstrated that any nonsingular hypersurface of CP^3 is almost completely decomposable. In this paper we first announce generalizations of this result in two directions as follows.

THEOREM 1. *Suppose W is a simply-connected nonsingular complex projective 3-fold. Then there exists an integer $m_0 \geqslant 1$ such that any hypersurface section V_m of W of degree $m \geqslant m_0$ which is nonsingular will be almost completely decomposable.*

THEOREM 2. *Let V be a nonsingular complex algebraic surface which is a complete intersection. Then V is almost completely decomposable.*

IDEA OF PROOF. The idea of the proofs is to degenerate V (or V_m) to a pair of "less complicated" nonsingular surfaces crossing transversely and then use induction. The topological analysis of such a situation is then taken care of by Corollary 2.5 of [**MM2**] which states:

COROLLARY. *Suppose W is a compact complex manifold and V, X_1, X_2 are closed complex submanifolds with normal crossing in W. Let $S = X_1 \cap X_2$ and $C = V \cap S$ and suppose as divisors V is linearly equivalent to $X_1 + X_2$. Let $\sigma\colon X_2' \longrightarrow X_2$ be the monoidal transformation of X_2 with center C. Let S' be the strict image of S in X_2' and let $T_2' \longrightarrow S'$, $T_1 \longrightarrow S$ be tubular neighborhoods of S' in X_2' and S in X_1, respectively, with $H_1 = \partial T_1$* and *$H_2' = \partial T_2'$.*

Then there exists a bundle isomorphism $\eta\colon \underline{H_2' \longrightarrow H_2}$ which reverses orientation on fibers such that V is diffeomorphic to $\overline{X_2' - T_2'} \cup_\eta \overline{X_1 - T_1}$.

Then if V, X_1, X_2 are simply connected 4-manifolds we can use [**M**] to

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AMS (*MOS*) *subject classifications* (1970). Primary 57D55, 57A15, 14J99.